# 9.) Magnetic forces

9.1 Magnetic forces on charged particles

9.2 Magnetic forces on currents

## 9.1) Magnetic forces on charged particles

(*) Key Points:

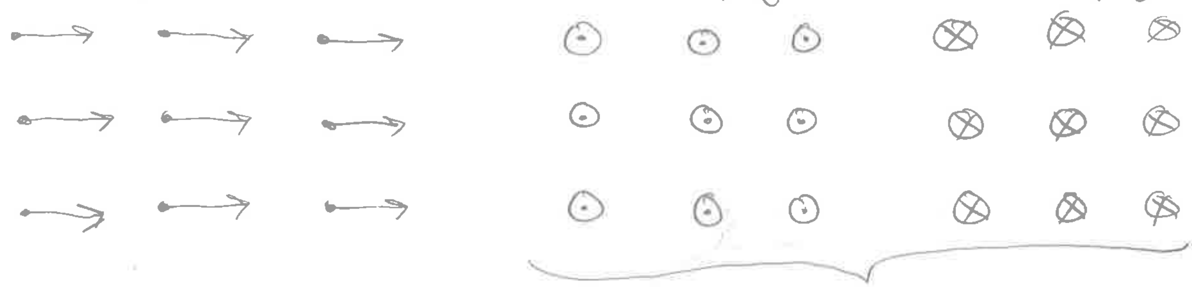
(a) All magnetic fields are produced by charges in motion (Earth's magnetic field, bar magnets, etc.)

(b) Magnetic forces act only on moving charges

First, let's assume a magnetic field exists in space (next chapter we will discuss origin of magnetic fields)

(*) Magnetic field $\vec{B}(\vec{x})$, like the electric field $\vec{E}(\vec{x})$, is a vector field (thinking in 3D will be crucial now)

Uniform $\vec{B}$   $\vec{B}$ out of page   $\vec{B}$ into page



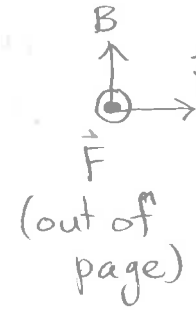
Imagine looking at tip or tail of an arrow

⊛ Unlike $\vec{E}$ field, the magnetic field is <u>not</u> parallal or antiparallel to magnetic force. Instead,

$$\vec{F}_B = q\,\vec{v} \times \vec{B} = qvB\sin\theta \quad \text{(right-hand rule for direction)}$$

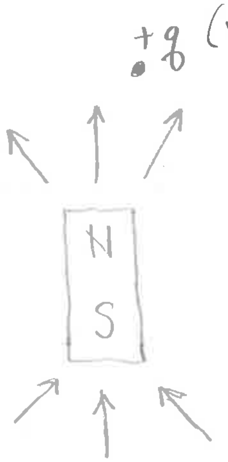
velocity of charge $q$

magnetic field at location of $q$

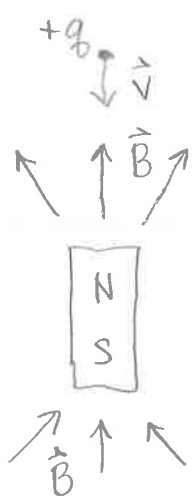




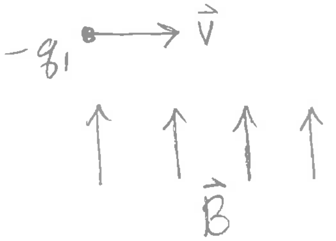
## Examples




$\vec{F}_q = 0$ since $q$ is stationary




$\vec{F}_q = 0$ since $\vec{v} \times \vec{B} = vB\sin\pi = 0$




$\vec{F}_q$ points into the page:

$\vec{v}\times\vec{B}$ (out of page)

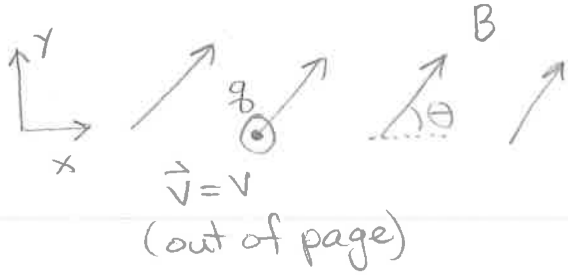

$\Downarrow$

$q\vec{v}\times\vec{B} = -q_1\vec{v}\times\vec{B}$ (into page)

(*) Electric and Magnetic forces add together independently, just like other forces:

$$\vec{F}_{total} = q(\vec{E} + \vec{v}\times\vec{B})$$

<u>Question</u>: What is the magnetic force acting on charge $q$ at the instant shown?




(a) $qvB\sin\theta\ \hat{\imath}_y$

(b) $qvB\sin\theta\ \hat{\imath}_x$

(c) $qvB\sin\theta$ $\nwarrow$

(d) $qvB\sin\theta$ $\searrow$

(e) $qvB$ $\nwarrow$

Answer: (e)

Note that the vector $\vec{v}$ is perpendicular to $\vec{B}$

$\Rightarrow F_q = qvB$ and the direction $\nwarrow$ comes from RHR

(*) Magnetic fields have another property that differs significantly from electric fields:

$$\boxed{\oint \vec{B}\cdot d\vec{A} = 0} \quad \text{(universal law)}$$

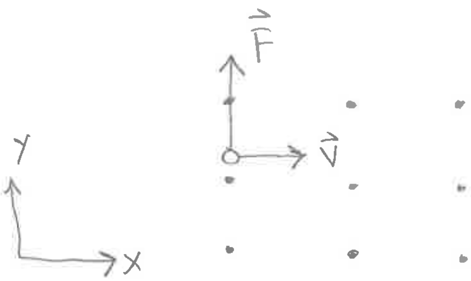
$\Rightarrow$ All magnetic fields close in on themselves, and the fundamental source of $\vec{B}$ fields are "magnetic dipoles"


Magnetic dipole

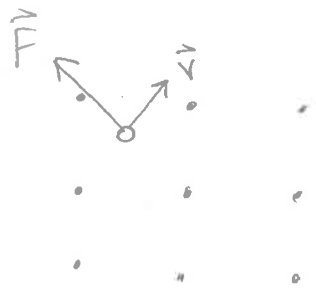
Electric dipole

Application: paths of charged particles in uniform $\vec{B}$ field

uniform $\vec{B}$ (out of page)




$$\vec{F} = -q\vec{v}\times\vec{B} = qvB\,\hat{i}_y$$




$\vec{F}$ always perpendicular to $\vec{v}$ and the path taken by the particle

$\Rightarrow$ circular path

$$\underbrace{m\frac{v^2}{R}}_{\text{Centripetal force}} = \underbrace{qvB}_{\text{magnetic force}}$$

$$\Rightarrow R = \frac{mv}{qB} \quad \text{(radius of circular path)}$$

Also, $W = \int \vec{F}\cdot d\vec{r} = 0$ (since $\vec{B}$ is always perpendicular to path)

(*) Magnetic forces do no work on moving charges!

## 9.2) Magnetic forces on current-carrying wires

⊛ Current in a wire is just a bunch of individual moving charges

⊛ Easy to derive force on current-carrying wire from fundamental law $\vec{F}_B = q\vec{v}\times\vec{B}$:

Current $\vec{i} = \rho_c A \vec{u}$ ← Recall $\rho_c$ is charge density, $A$ is cross-sectional area, $u$ is drift speed.

$\Rightarrow$ average velocity $\vec{u} = \dfrac{\vec{i}}{\rho_c A}$

$$\Rightarrow F_B = q\left(\frac{\vec{i}}{\rho_c A}\right)\times\vec{B} \quad \text{(force on \underline{one} charge } q\text{)}$$

$$F_B^{total} = Nq\left[\frac{\vec{i}}{\rho_c A}\times\vec{B}\right] \quad \text{(force on N charges)}$$

$$= (Nq)\left[\frac{\vec{i}\times\vec{B}}{(Nq/V)A}\right]$$

$$= \frac{\vec{i}\times\vec{B}}{A/V} \qquad (V = A\ell)$$

$$= \ell\,\vec{i}\times\vec{B} \quad (\ell \text{ is length of wire})$$

$$\boxed{\vec{F}_B^{total} = i\,\vec{\ell}\times\vec{B}} \quad (\vec{i} \text{ and } \vec{\ell} \text{ are parallel})$$

⇑

Net force on straight wire in uniform $\vec{B}$ field

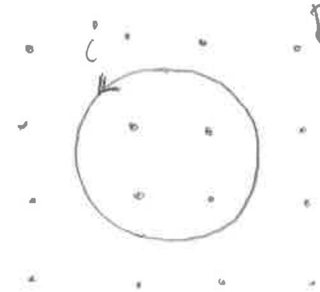
⊛ If wire bends or $\vec{B}$ field changes along length of wire, then we need to integrate:

$$d\vec{F} = i\, d\vec{l} \times \vec{B}$$

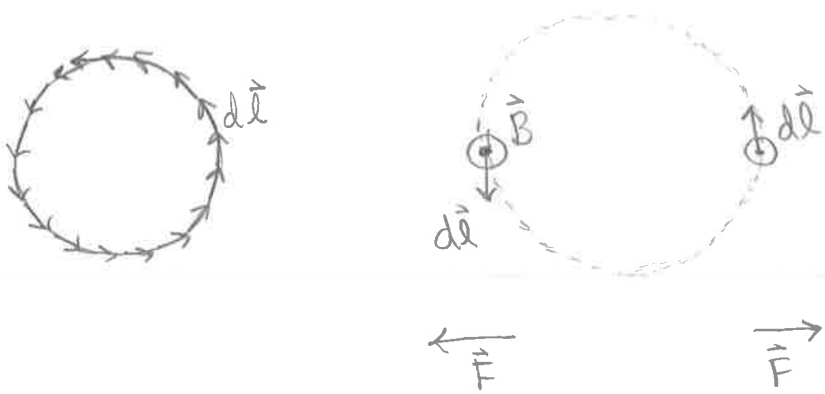
Question: What is the net force on a circular wire loop carrying current $i$ that is placed in a perpendicular uniform $\vec{B}$ field?

Solution: Consider the figure to the right.

Break up wire into small segments $d\vec{l}$



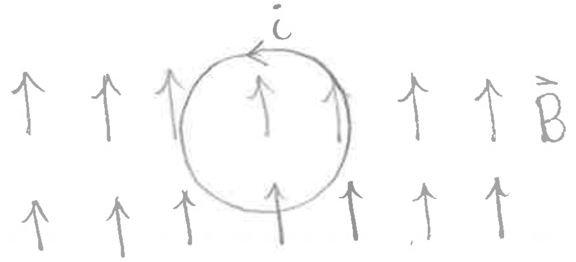

$\vec{B}$ out of page

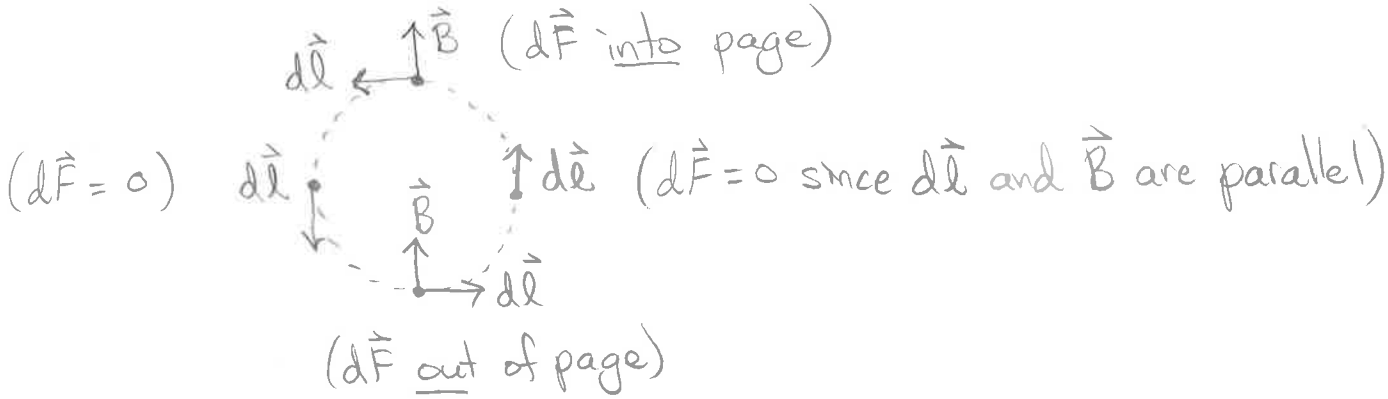



⊛ Opposite sides of the circle experience equal and opposite forces $\Rightarrow$ no net force on wire.

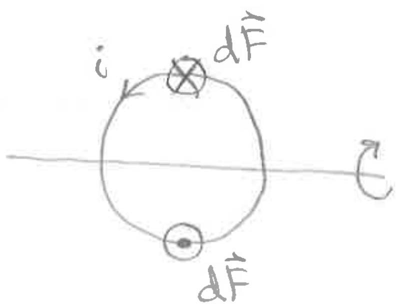
Question: what if instead the $\vec{B}$ field were parallel to the plane of the wire?

Solution:



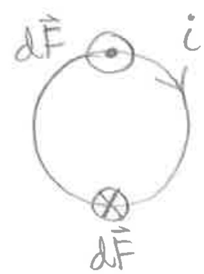




(*) In this case the loop experiences no net force, but it does experience a torque that causes the loop to rotate



Does the loop Keep increasing its rotation rate?

Answer: No, once the loop has rotated 180° the current is now opposite and the torque will reverse.




(*) An electric motor takes advantage of this by switching the current direction after 180° so that torque remains in same direction continuously

$\Rightarrow$ Electrical energy in battery converted to mechanical energy of rotation.