PHYS 206 Lecture 7

Outline

1. Momentum
2. Conservation of momentum
3. Collisions: Elastic and inelastic

Definition (colloquial): Momentum describes how difficult it is to stop an object along its path.

Example: The Texas A&M football team has a lot of momentum going into the second half.

In terms of an object's properties, it is difficult to stop fast-moving objects and massive objects.

Definition: $\vec{p} = m\vec{v}$ the momentum is a vector quantity equal to mass × velocity.

You might be thinking: "What is the point? Why introduce a new term that is very similar to velocity? I just multiply by the mass?"

No! Momentum is a much more useful concept than velocity and in some sense it is more fundamental.

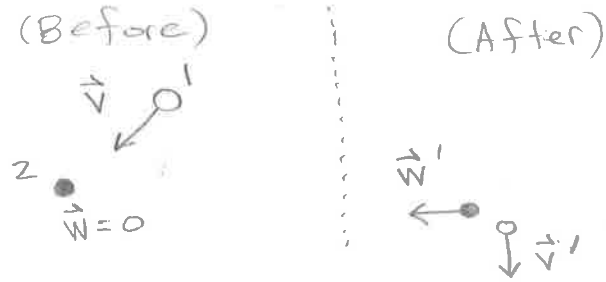
Why?

a) Momentum is conserved (velocity is not conserved)

Previously: Energy conservation was found to be a powerful law underlying the motion of physical objects.

Later in course: Angular momentum conservation will be a powerful tool to understand rotating objects' motion.

Example: Two billiard balls collide on a pool table



Momentum vector before is equal to momentum vector after.

Before: $\vec{P}_{tot} = M_1 v_x \hat{i}_x + M_1 v_y \hat{i}_y$

After: $\vec{P}_{tot} = (M_1 v_x' + M_2 w_x')\hat{i}_x + (M_1 v_y' + M_2 w_y')\hat{i}_y$

The momentum vector is conserved (not just its magnitude). Therefore, the components of $\vec{P}$ are individually conserved:

$$\left.\begin{aligned} x:&\quad M_1 v_x = M_1 v_x' + M_2 w_x' \\ y:&\quad M_1 v_y = M_1 v_y' + M_2 w_y' \end{aligned}\right\}$$

Seems complicated! Two equations and 4 unknowns (v_x', v_y', w_x', w_y')

(*) Important point: Momentum conservation alone cannot tell us everything about the final trajectories.

That's okay, because there are different <u>types</u> of collisions:

(1) Elastic: Kinetic energy conserved

(2) Inelastic: Kinetic energy <u>not</u> conserved

↳ An important type of inelastic collision is one in which the objects move away while attached.

⊗ Depending on whether collision is elastic or inelastic, we can obtain a sufficient number of equations to solve for the motion.

<u>Example</u>: Suppose billiard ball collision is elastic. Then we have one more equation

$$\frac{1}{2}M_1V^2 = \frac{1}{2}M_1V'^2 + \frac{1}{2}M_2W'^2$$

Initial Kinetic energy is equal to final Kinetic energy.

$$\Rightarrow \frac{1}{2}M_1(V_x^2+V_y^2) = \frac{1}{2}M_1(V_x'^2+V_y'^2) + \frac{1}{2}M_2(W_x'^2+W_y'^2)$$

Still need <u>one</u> more equation!

Note this is physically meaningful.

Depending on whether collision is head-on or a glancing collision, the balls will leave at different angles. So, to solve equations <u>we need to tell you</u> something about final directions of motion.

For one-dimensional problems, knowing that the collision is elastic gives us everything

$$M_1 v = M_1 v' + M_2 w' \quad \underline{\text{and}} \quad \tfrac{1}{2} M_1 v^2 = \tfrac{1}{2} M_1 v'^2 + \tfrac{1}{2} M_2 w'^2$$

$$w' = \frac{M_1 (v - v')}{M_2}$$

$$\Rightarrow \tfrac{1}{2} M_1 v^2 = \tfrac{1}{2} M_1 v'^2 + \tfrac{1}{2} \frac{M_1^2}{M_2} (v - v')^2$$

Solve this quadratic equation as homework.

What if $M_1 = M_2$?

Then $v^2 = v'^2 + v^2 - 2vv' + v'^2$

$$\Rightarrow 0 = 2v'^2 - 2vv' \Rightarrow v'(v - v') = 0$$

Do <u>not</u> divide both sides by v' !

$\underline{v' = 0}$ or $\underline{\underline{v = v'}}$ are solutions.

then $\underline{w' = v}$ or $\underline{w' = 0}$.

Weird. Physically we know that $\{v = v' \text{ and } w' = 0\}$ can't be a solution. The one billiard ball would just pass through the second ball.

But this is always a trivial solution to the conservation of momentum and conservation of kinetic energy equations.

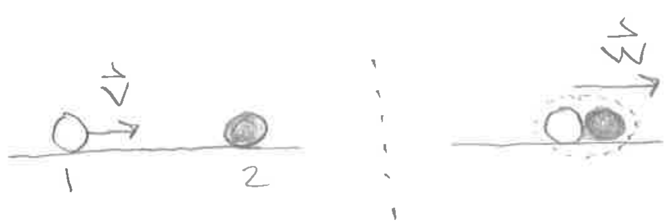
Real-life examples:

(a) Photons "passing through" transparent glass

(b) Neutrinos passing through Earth

For this class always ignore these types of solutions.

Inelastic collision example:



One object is wax, so the two objects connect and move away joined.

In this case energy is lost in deforming the wax.

⊛ Kinetic energy not conserved in inelastic collisions.

Solution: $P_i = M_1 V$

$P_f = (M_1+M_2) W$

$$\Rightarrow \quad M_1 V = (M_1+M_2) W$$

$$\Rightarrow \quad W = \frac{M_1}{M_1+M_2} V$$

⊛ Momentum conservation alone solves the question.

Many different variations on this theme

(a) Bullet shot into piece of wood

(b) Two cars colliding and moving away together in a connected single piece

How much energy is "lost" in the example above?

Initial kinetic energy: $KE_i = \frac{1}{2} M_1 v^2$

Final kinetic energy: $KE_f = \frac{1}{2}(M_1+M_2) w^2$

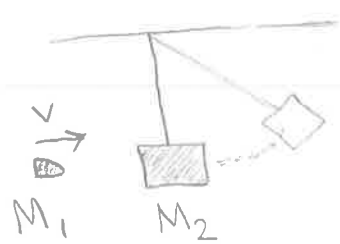
$$= \frac{1}{2}(M_1+M_2)\left[\frac{M_1^2}{(M_1+M_2)^2} v^2\right]$$

$$= \frac{1}{2}\frac{M_1^2}{M_1+M_2} v^2$$

$$\Delta KE = KE_f - KE_i = \left(\frac{1}{2}M_1 - \frac{1}{2}\frac{M_1^2}{M_1+M_2}\right) v^2$$

$$= \frac{1}{2}\left(\frac{-M_1 M_2}{M_1+M_2}\right) v^2$$

Another example: Bullet fired into block. How high does block go?



We saw that $KE_f = \frac{1}{2}\frac{M_1^2}{M_1+M_2} v^2$.

At max angle of deflection this kinetic energy is converted into gravitational potential energy:

$$\frac{1}{2}\frac{M_1^2}{M_1+M_2} v^2 = (M_1+M_2) g H \Rightarrow H = \frac{1}{2g}\left(\frac{M_1}{M_1+M_2}\right)^2 v^2$$

5 Steps for solving conservation of momentum:

① Identify a momentum conservation question by looking for a collision or explosion that involves multiple masses.

② Write down the Law of Conservation of Momentum:

One-dimensional	Two-dimensional
$\vec{p}^{\,i}_{total} = \vec{p}^{\,f}_{total}$	$\vec{p}^{\,i}_{total} = \vec{p}^{\,f}_{total}$
$p^{i}_{total} = p^{f}_{total}$	$\left\{ \begin{array}{l} p^{i}_{x,total} = p^{f}_{x,total} \\ p^{i}_{y,total} = p^{f}_{y,total} \end{array} \right\}$

③ Calculate the individual momenta $\vec{p} = m\vec{v}$ of each object involved both before and after the collision. Always carefully consider the sign of each term.

④ Add up all momenta before collision and all momenta after collision. Set equal.

⑤ Be careful if some other object blocks part of the motion → cannot use conservation of momentum.

Why is momentum conserved?

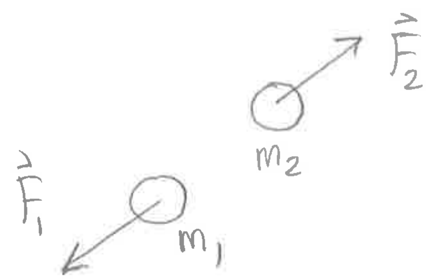
→ Rewrite Newton's 2nd Law : $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$

$$\Rightarrow \vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$ ← Alternate form of 2nd Law.

$\Rightarrow d\vec{p} = \vec{F}dt$ over a small time interval dt.

Now consider two object's interacting via some force $\vec{F}$.

From Newton's 3rd Law, $\vec{F}_1 = -\vec{F}_2$



Over a small interval of time,

$$d\vec{p}_1 = \vec{F}_1 dt \quad \text{and} \quad d\vec{p}_2 = \vec{F}_2 dt = -\vec{F}_1 dt = -d\vec{p}_1$$

⊛ The change in momentum of object 1 is equal and opposite to change in momentum of object 2.

Therefore, total change in momentum of combined system is $d\vec{p}_1 + d\vec{p}_2 = d\vec{p}_1 - d\vec{p}_2 = 0$.