7.) Current and Ohm's Law

7.1) Current
7.2) Ohm's Law
7.3) Resistors

In other words, "What's going on inside conductors in a circuit?"

7.1) Current

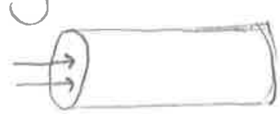
⊛ A conducting wire connected to <u>both</u> ends of a battery gives charges a path to lower their potential energy

<u>Definition</u>: The flow of this charge from high potential energy to low potential energy is called "current":

$$I = \frac{dQ}{dt} \quad \left[\text{units of Amperes} = \frac{C}{s}\right]$$

⊛ Think of $Q(t)$ as being the total amount of charge that has left the battery.

⊛ Alternatively, current is like a flux of charge through the cross-sectional area of a conducting wire



$$\text{Air Flux} = \vec{u}\cdot\vec{A} = \frac{dV}{dt} \quad \begin{pmatrix} V = \text{volume of air} \\ \vec{u} = \text{velocity field of air} \end{pmatrix}$$

$$\text{Charge Flux} = \frac{dQ}{dt} = \frac{d(\rho_c V)}{dt} = \rho_c \frac{dV}{dt} = nq\,\vec{u}\cdot\vec{A}$$

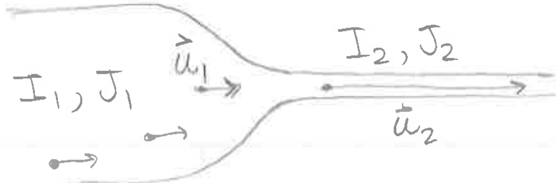
$$\left(\begin{array}{l} q = \text{charge of mobile particles} \\ \rho_c = \text{volume charge density} = \dfrac{Nq}{V} \\ n = \text{number density of charge carriers} = \dfrac{N}{V} \\ N = \text{total number of charge carries} \end{array}\right)$$

A related and very important concept is the "current density"

$$J = \frac{I}{A} = \rho_c u \quad (\text{bulk form})$$

$$dI = J\,dA \quad (\text{infinitesimal form})$$

⊛ Very important point: For two sections of a circuit in series, current must be identical but current density can change

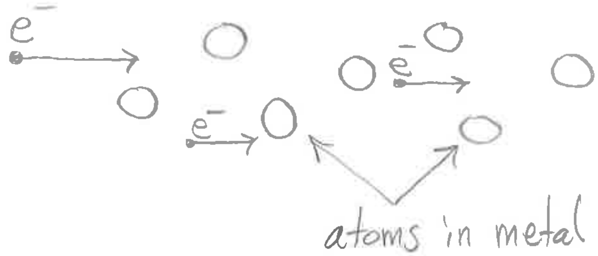


$$\left[\begin{array}{c} I_1 = I_2 \\ A_1 > A_2 \\ J_1 = \dfrac{I_1}{A_1} < J_2 = \dfrac{I_2}{A_2} \end{array}\right]$$

⊛ Think of partially blocking the end of a water hose

→ water shoots out because current must be conserved $(\vec{u}_2 > \vec{u}_1)$

The velocity $\vec{u}$ of the charge carriers is like a "terminal velocity"



Electrons constantly run into obstacles and only have an average "drift velocity"

Electric field produced by battery accelerates electrons until a collision takes place, then must accelerate again.

7.2) Ohm's Law

Suppose the electric field in the two conductors below is the same ... which has the greater current?

Two conductors. Same material.

Current through (1) should be larger since there are more mobile charges present to accelerate.

What about the two drift velocities $\vec{u}_1$ and $\vec{u}_2$?

Answer: Should be identical since $\vec{E}$ accelerates the electrons equally: $\vec{u} = \alpha \vec{E}$ (α a proportionality factor)

$\Rightarrow$ Therefore the current densities are identical

$$\left.\begin{aligned} J_1 &= \rho_c u_1 = \rho_c(\alpha E) \\ J_2 &= \rho_c u_2 = \rho_c(\alpha E) \end{aligned}\right\}$$ Here α is just the constant of proportionality

(*) However different conductors have different ρ_c and can also give rise to different $\vec{u}$ (more or less obstacles for electrons to run into).

(*) Both are encoded in the "resistivity" (ρ) of the material.

General Law:

$$\boxed{\vec{J} = \frac{1}{\rho}\vec{E} \quad \text{Ohm's Law}}$$

Charge density denoted by ρ_c above so no confusion with resistivity ρ.

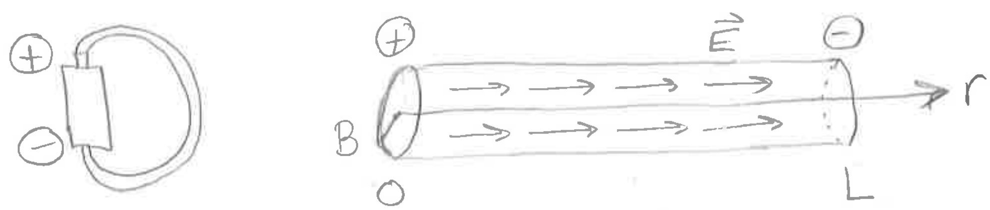
(*) Current density grows linearly with the applied electric field

(*) Ohm's Law can break down for large electric fields, but we will neglect that possibility

(*) Resistivity independent of geometry $\rightarrow$ only depends on material (gold, copper, etc.) and the temperature

$$\rho(T) = \rho_0\left[1 + \alpha(T - T_0)\right]$$

Example: Consider a circular wire of radius B and length L made from a material with resistivity ρ. If the wire is connected to a battery with voltage V, what current flows?



Solution: Since the cross sectional area πB^2 is constant, the current density $J = \frac{I}{A} = \frac{I}{\pi B^2}$ is constant. From Ohm's Law $\vec{E} = \rho \vec{J}$ must also be constant.

$$\Rightarrow V = V(+) - V(-) = -\int_L^0 \vec{E} \cdot d\vec{r} = -\int_L^0 E\, dr = EL$$

Voltage across battery

why are bounds not $\int_0^L$??) Because $\oplus$ terminal is at $r = 0$.

$$\Rightarrow V = EL = (\rho J)L = \rho\left(\frac{I}{\pi B^2}\right)L$$

$$\Rightarrow \underbrace{\left(\rho \frac{L}{\pi B^2}\right)}_{\equiv \text{"Resistance" } (R)} I$$

(*) We have proven that the induced current is proportional to the applied voltage

Generally: $\boxed{\Delta V = IR}$ where $\boxed{R = \rho \frac{L}{A}}$ cross sectional area

(Ohm's Law variation)

(*) Resistance increases for longer wires and decreases for wider (area) wires

7.3) Resistors

Resistors come in many different shapes, sizes, and materials.

(*) Need to be able to apply Ohm's Law for different situations.

Follow these 5 steps:

(1) Identify the direction of current i and label it "r".

(2) Determine the cross sectional area A that is perpendicular to the current flow.

(3) Compute the current density $J = \frac{i}{A}$.

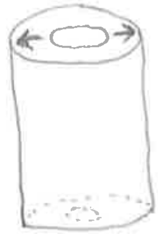
(4) Find $\vec{E}$ from $\vec{J}$ using Ohm's Law: $\vec{E} = \rho \vec{J}$.

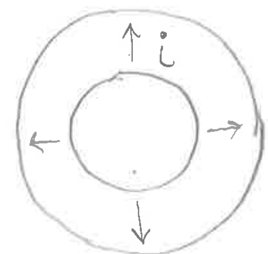
(5) From $\vec{E}$ compute potential difference $\Delta V = -\int \vec{E} \cdot d\vec{r}$.

Example: Suppose there is a hollow conducting cylinder with inner radius a and outer radius b connected to a battery so that a current i flows radially outward. The cylinder has resistivity ρ and height H. Find the current density, $V(a) - V(b)$, and the cylinder's resistance R.

Solution: Follow 5 steps above.

(1) Direction of current flow is radially outward





(overhead)

(2) Cross-sectional area depends on r in this case: $\underline{A(r) = 2\pi r H}$

(Note that if current had been flowing along H direction, then $A = \pi(b^2 - a^2)$ would have been constant)

(3) Current density is therefore $J = \frac{i}{A} = \frac{i}{2\pi r H}$

(4) Electric field is therefore $E = \rho J = \rho \frac{i}{2\pi r H}$

(5) Potential difference $V(a) - V(b) = -\int_b^a \vec{E} \cdot d\vec{r}$

$$= -\int_b^a \left(\rho \frac{i}{2\pi r H} \hat{i}_r \right) \cdot \left(dr\, \hat{i}_r + r\, d\theta\, \hat{i}_\theta \right)$$

$$= -\frac{\rho i}{2\pi H} \int_b^a \frac{1}{r}\, dr$$

$$= -\frac{\rho i}{2\pi H} \left[\ln (a/b) \right]$$

$$= \frac{\rho i}{2\pi H} \ln (b/a) = \left[\frac{\rho}{2\pi H} \ln (b/a) \right] i \equiv R i$$

Finally, the resistance can be obtained from

$$\Delta V = i R$$

$$\Rightarrow \frac{\rho i}{2\pi H} \ln (b/a) = i R$$

$$\Rightarrow \boxed{R = \frac{\rho}{2\pi H} \ln (b/a)}$$

Notice the "opposite" behavior of capacitors and resistors:

Capacitors $\rightarrow \Delta V \sim Q \Rightarrow \Delta V = \frac{1}{C} Q \Rightarrow C = \frac{Q}{\Delta V}$

Resistors $\rightarrow \Delta V \sim I \Rightarrow \Delta V = RI \Rightarrow R = \frac{\Delta V}{I}$

* Large capacitance $\rightarrow$ large charge
* Large resistance $\rightarrow$ small current

Resistors behave opposite compared to capacitors in series and parallel:

Parallel: $C_{eq} = C_1 + C_2$ $\qquad R_{eq} = \frac{1}{R_1} + \frac{1}{R_2}$

Series: $C_{eq} = \frac{1}{C_1} + \frac{1}{C_2}$ $\qquad R_{eq} = R_1 + R_2$