6.) Capacitors

6.1) Electric field between charged conductors

6.2) Capacitors

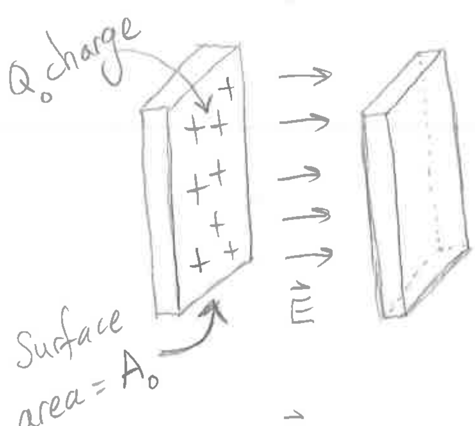
6.3) Capacitors in circuits

6.1) Electric field between charged conductors

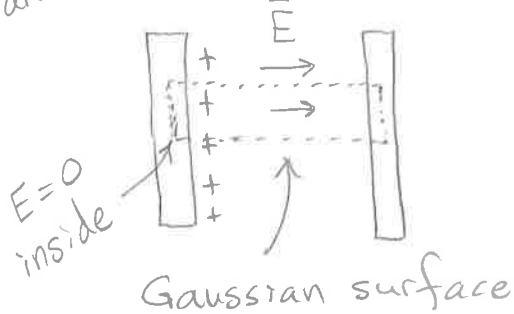
(*) Let's use Gauss's Law to derive some interesting features of conductors

Example: Consider two conducting plates near each other.

If one of them has a surface charge density $\sigma = \frac{Q_0}{A_0}$, what is the charge density on the other?



The electric field between the two plates will be nearly horizontal (except at the edges) by symmetry.

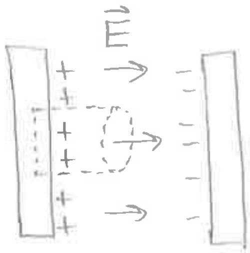


Draw Gaussian surface as shown.

(*) No electric flux through this surface!

$\Rightarrow$ $\boxed{\text{Equal and opposite charge on right plate}}$

(*) What about $\vec{E}$ field between plates?



Draw a cylindrical Gaussian surface of radius R and length L.

$\vec{E}$ is perpendicular and constant along circular face (translational symmetry)

$$\Rightarrow \oint \vec{E}\cdot d\vec{A} = E\int dA = E(\pi R^2)$$

$$q_{enc} = \int dq = \int \sigma\, dA = \int \left(\frac{Q_0}{A_0}\right) dA$$

$$= \frac{Q_0}{A_0} A = \frac{Q_0}{A_0}(\pi R^2)$$

Total area of conducting plate (A_0)

area of Gaussian surface (A)

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow E(\cancel{\pi R^2}) = \frac{1}{\varepsilon_0}\frac{Q_0}{A_0}(\cancel{\pi R^2})$$

$$E = \frac{1}{\varepsilon_0}\frac{Q_0}{A_0}$$

$$\boxed{E = \frac{1}{\varepsilon_0}\sigma}$$

Independent of L!

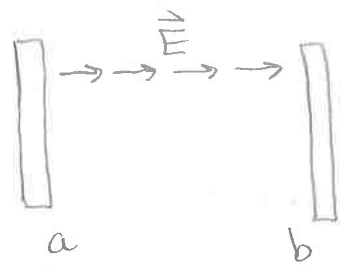
(*) Electric field is constant as you move from one plate to the other

Follow up question: what is the potential difference between the two plates?

Solution: Easy since $\vec{E}$ is constant...

$\vec{E} = E\,\hat{i}_x$

$d\vec{r} = dx\,\hat{i}_x + dy\,\hat{i}_y$



a at higher electric potential than b

$$V(a) - V(b) = -\int_b^a \vec{E}\cdot d\vec{r}$$

$$= -\int_b^a E\,dx = -E\int_b^a dx$$

$$= -E(a-b)$$

$$= \frac{\sigma}{\varepsilon_0}(b-a) = \boxed{\frac{Q_0}{A_0\varepsilon_0}(b-a)}$$

(*) Note that $\Delta V = V(a) - V(b)$ is proportional to Q_0 (the total charge on the positive conductor)

This will always be true, even for different geometries

$\Rightarrow$ Leads to definition of capacitance.

6.2) Capacitors

* Definition: A capacitor consists of two equal but oppositely-charged conductors separated by an insulator (such as air).

* Capacitors are completely characterized by a quantity called the "capacitance" C, defined as

$$\boxed{C \equiv \frac{Q}{\Delta V}}$$

Different geometries can give rise to same capacitance. They behave the same in circuits.

* Q is the total charge on the positive conductor.
* ΔV is the potential difference between the two conductors

Note that ΔV will always be proportional to Q and therefore C depends only on the geometry of the conductors and not on the actual charge Q on the conductors.

* Physically, a large capacitance means that a lot of charge can be put on the conductors for a given value of ΔV.

To compute the capacitance for any geometry:

1. Assume there is a charge $\pm Q$ on the two conductors
2. Compute $\vec{E}$ in the region between the conductors (using Gauss's Law)
3. From $\vec{E}$, calculate $\Delta V = V(+) - V(-) = -\int_{-Q}^{+Q} \vec{E} \cdot d\vec{r}$

   (this choice will always give a positive ΔV, since positive charges are located at higher potential)
4. Finally, compute $C \equiv \dfrac{Q}{\Delta V}$ (Q will always drop out)

6.3) Capacitors in circuits

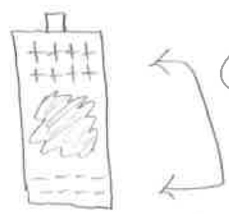
* This begins our discussion of circuits
* We have already derived the fundamental physics laws needed to study many (but not all circuits)

Let's introduce two "circuit elements" (out of 4 total for this class)

1. Batteries
2. Capacitors

- Fundamentally, a battery is a device that maintains a <u>constant</u> electric potential between its + and − terminals

- A battery is able to maintain the separation of + and − charges through chemical reactions



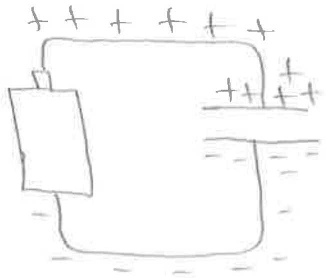
+ and − charges are attracted, but chemical reactions prevent them from moving <u>through</u> the battery <u>interior</u>

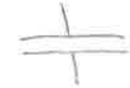
However, if you connect the terminals by a conductor, charge <u>can</u> flow the long way around

- Regardless of how fast charge flows out of a battery, the potential difference <u>ΔV</u> <u>stays</u> <u>the</u> <u>same</u>

(As soon as one of the charges makes it to the other terminal, a chemical reaction in the battery will push it back through the battery to where it started ... Kind of futile!)

Question: What if we don't allow the charges to complete the circuit by inserting a capacitor?



Capacitor symbol 

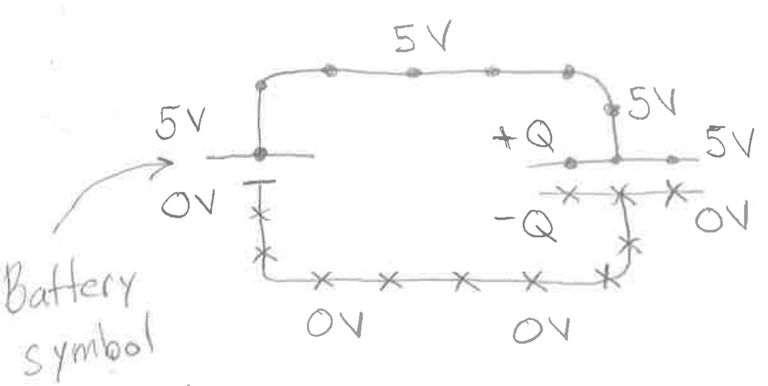
insulator (air) between plates, charge cannot flow across!

Charge will flow until the ΔV across capacitor matches the ΔV of the battery.

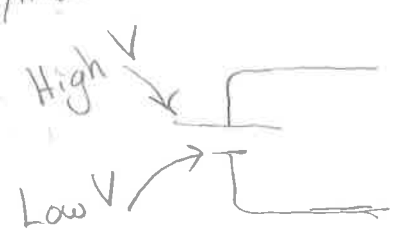
Why??

Answer: In electrostatics conductors have $\vec{E} = 0$ inside $\Rightarrow$ conductors are equipotential surfaces

$$\Delta V = -\int \vec{E} \cdot d\vec{r} = 0$$



Charge $\boxed{Q = C\,\Delta V}$ builds up on the two sides of the capacitor.

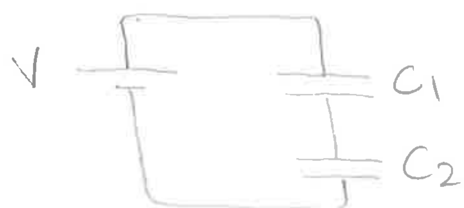


(*) Analyze multiple capacitors in circuits by remembering

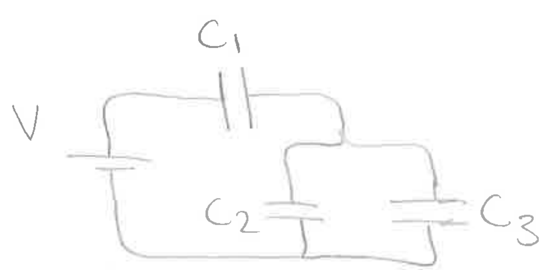
1. Wires are equipotential surfaces
2. Capacitors hold equal and opposite charge $\pm Q$

Capacitors in series

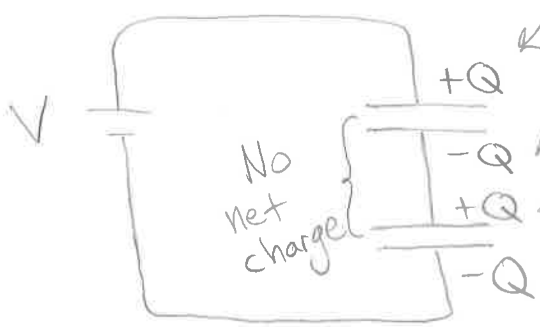
(*) By "series" we mean that there are no optional routes between the two elements:



All 3 circuit elements (1 battery, 2 capacitors) are in "series".



Only battery V and capacitor C_1 are in series.

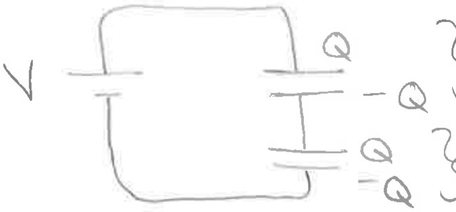


Assume +Q builds up on top plate

−Q on opposite side of capacitor C_1

Inner conductor has no net charge, so +Q remains

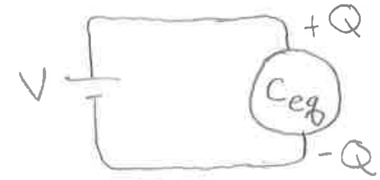
Finally −Q on opposite side of capacitor C_2



$\Delta V_1 = \frac{Q}{C_1}$

$\Delta V_2 = \frac{Q}{C_2}$

$\approx$



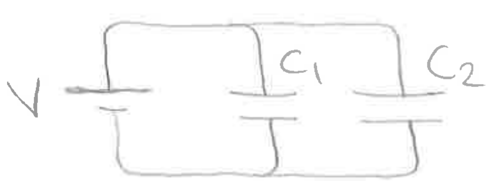
$$V = \Delta V_1 + \Delta V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \equiv \frac{Q}{C_{eq}}$$

$$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Two capacitors in series behave like a single capacitor with equivalent capacitance C_{eq}.

Capacitors in parallel

(*) Two circuit elements are in "parallel" if each pair of ends are connected by only a conducting wire



All 3 elements (V, C_1, C_2) are in parallel

Only V and C_1 are in parallel

⊛ All elements in parallel have the same potential difference across them (the conducting wires connecting their ends are at the same potential)

$$\Delta V_1 = \frac{Q_1}{C_1} \Rightarrow Q_1 = C_1 V \qquad \Delta V_2 = \frac{Q_2}{C_2} \Rightarrow Q_2 = C_2 V$$

Total charge on top plates $Q_{tot} = Q_1 + Q_2$

$$\Rightarrow \quad C_{eq} \equiv \frac{Q_{tot}}{\Delta V} = \frac{Q_1 + Q_2}{V} = \frac{C_1 V + C_2 V}{V}$$

$$\Rightarrow \quad \boxed{C_{eq} = C_1 + C_2}$$

Capacitors in parallel behave like a single capacitor with equivalent capacitance C_{eq}.

⊛ Energy stored in capacitor

$$\boxed{W = \frac{1}{2} C V^2}$$