PHYS 206 Lecture 5

Outline

5.1) Energy
5.2) Work
5.3) Vector dot product

* Very often we don't know all the forces acting on an object.

Examples: Shoot a bullet into a block of wood
Safety crash testing of automobiles

Nevertheless, we can obtain information about the motion by using conservation laws.

A "conserved" quantity is one that remains unchanged throughout the motion

Examples: (1) Playing poker: the chips move from one player to another, but the total value of all chips is constant during the game.

(2) Total number of people in the world: not conserved.

(3) Paying for dinner with a credit card: total amount of money between you and restaurant is not conserved $\longrightarrow$ credit card company takes a small percentage. But expand the system to include CC company, and now money is conserved.

There are physics analogs of these three examples (discuss later)

First, what quantities are conserved in physics?

(1) Energy

(2) Momentum → later in course

(3) Angular Momentum → later in course

Why are certain quantities conserved?

→ Very deep question!

Ultimately due to symmetry of universe

(1) Translation in time (energy conservation)

(2) Translation in space (momentum conservation)

(3) Rotational invariance (angular momentum conservation)

Right now we are concerned with energy conservation.

Forms of energy you are familiar with:

(a) Kinetic energy - faster objects and more massive objects have the ability to do more damage

$$E_{Kin} = \frac{1}{2} M v^2$$

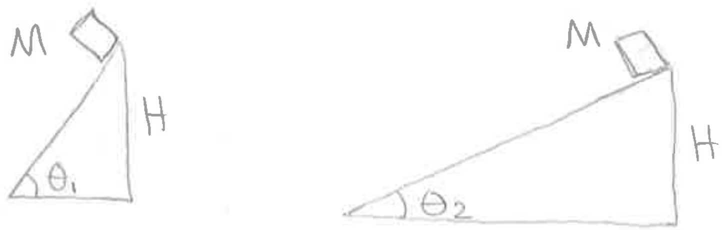
↑ Why the $\frac{1}{2}$?

(b) Potential energy - when dropped, more massive objects falling from greater heights have ability to do more damage

$$E_{pot}^{grav} = M g H$$

(c) other forms of energy?

Example: Consider two equal-mass blocks dropped from equal heights on two different ramps



What are their velocities at the bottom of ramps?

$$F_{1x} = Mg\sin\theta_1 = Ma_1 \Rightarrow a_1 = g\sin\theta_1$$

$$\Rightarrow v_1^2(t) = v_1^2(0) + 2a_1(x(t) - x(0)) = 0 + 2g\cancel{\sin\theta_1}\left(\frac{H}{\cancel{\sin\theta_1}}\right)$$

$$= 2gH$$

$$F_{2x} = Mg\sin\theta_2 = Ma_2 \ \ldots$$

$$v_2^2(t) = 2gH$$

Final velocity depends only on height, not angle.

Potential energy (gravitational) $= E_{pot}^{grav} = MgH$

Kinetic energy $= E_{Kin} = \frac{1}{2}Mv^2 = \frac{1}{2}M(2gH) = MgH$

Same

With these two definitions of E_{pot}^{grav} and E_{Kin}, it seems that energy is conserved for the falling blocks.

What if there is friction between blocks and ramps?

$$F_{1x} = Mg\sin\theta_1 - \mu Mg\cos\theta_1 = Ma_1$$

$$\Rightarrow v_1^2(t) = 2g(\sin\theta_1 - \mu\cos\theta_1)\left(\frac{H}{\sin\theta_1}\right) = 2gH - 2gH\mu\cot\theta_1$$

$$\text{vs. } v_2^2(t) = 2g(\sin\theta_2 - \mu\cos\theta_2)\left(\frac{H}{\sin\theta_2}\right) = 2gH - 2gH\mu\cot\theta_2$$

Friction reduces the final kinetic energy (like the credit card example above). That energy is not lost, but it is redistributed in the system in a nontrivial way.

Energy is closely associated with the concept of "work". Sometimes energy is defined as "the ability to do work."

Forces do work on objects:

$$\boxed{W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}}$$

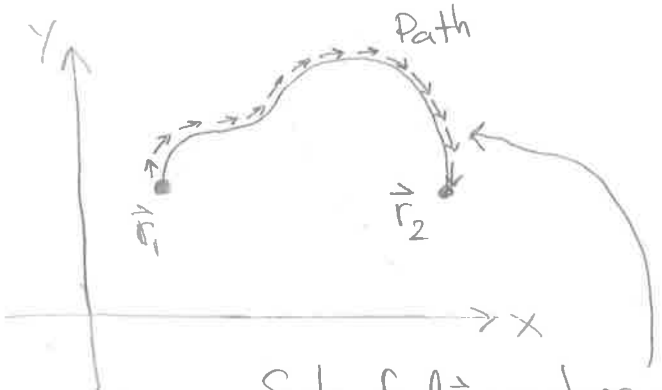
$d\vec{r} = dx\,\hat{i}_x + dy\,\hat{i}_y$

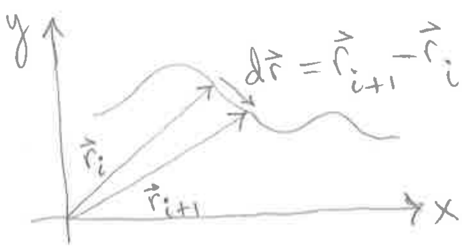
Vector "dot product"

New type of integral

"Line Integral"

Previously we have only considered integrals over time but work involves an integral over space, specifically over the path an object takes from $\vec{r}_1$ to $\vec{r}_2$.





Set of $d\vec{r}$ vectors making up the path

What about the vector dot product?

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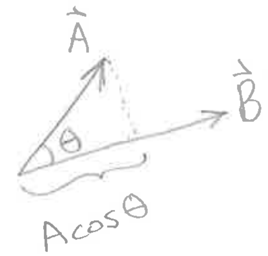
Since vectors involve direction, it is not obvious how to "multiply" them.

One way is the "dot product".

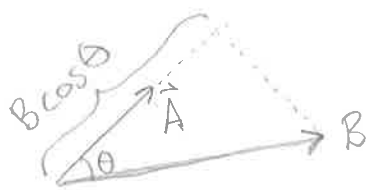
Geometric definition:

Project $\vec{A}$ onto $\vec{B}$



$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Project $\vec{B}$ onto $\vec{A}$



$$\vec{B}\cdot\vec{A} = AB\cos\theta$$

Result is a <u>number</u> not a vector.

If $\vec{A}$ and $\vec{B}$ are <u>perpendicular</u>, then $\vec{A}\cdot\vec{B} = 0$.

If $\vec{A}$ and $\vec{B}$ are <u>parallel</u>, then $\vec{A}\cdot\vec{B} = AB$.

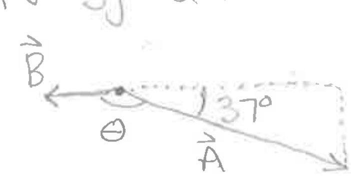
How to calculate $\vec{A}\cdot\vec{B}$ if the components are known:

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y \quad (2D)$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z \quad (3D)$$

<u>Example</u>: $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = -2\hat{i}$

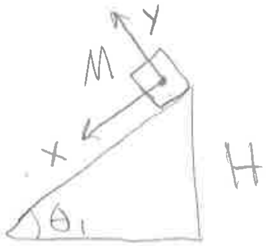
Solution 1:



$$(B)(A\cos\theta) = (2)(5\cos 143^\circ)$$
$$= (2)(-4) = -8$$

Solution 2 : $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (4)(-2) + (-3)(0) = -8$

Work done by friction in the ramp problem above :



$$W_f = \int \vec{F}_f \cdot d\vec{r}$$

the pathlength along x

$$= \int_{0}^{H/\sin\theta} (-\mu M g \cos\theta\, \hat{\imath}_x) \cdot (dx\, \hat{\imath}_x + dy\, \hat{\imath}_y)$$

starting position

$$= (-\mu M g \cos\theta_1)\left(\frac{H}{\sin\theta_1}\right)$$

$$= \underline{-\mu H M g \cot\theta_1}$$

Note that
$\hat{\imath}_x \cdot \hat{\imath}_x = 1$
$\hat{\imath}_x \cdot \hat{\imath}_y = 0$

Recall that the change in Kinetic energy was

$$KE(t) = \frac{1}{2} M v^2(t) = \frac{1}{2} M \left[2gH - 2gH\mu\cot\theta_1 \right]$$

$$= MgH - \underline{\mu H M g \cot\theta_1}$$

Apparently, the energy lost from friction is equal to the work that it did.

General result (Work-Energy Theorem)

$$\boxed{\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{tot} \cdot d\vec{r} = \frac{1}{2} M v_2^2 - \frac{1}{2} M v_1^2}$$

velocity at $\vec{r}_2$ Velocity at $\vec{r}_1$

The total force acting on the block-ramp system is

$$F_{tot} = Mg\sin\theta - \mu Mg\cos\theta$$

$$\Rightarrow \int_0^{H/\sin\theta} \vec{F}_{tot}\cdot d\vec{r} = \int_0^{H/\sin\theta} (Mg\sin\theta - \mu Mg\cos\theta)\, dr$$

$$= (Mg\sin\theta - \mu Mg\cos\theta)\frac{H}{\sin\theta}$$

$$= MgH - MgH\mu\cot\theta$$ (same as we found previously)

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Outline

5.4) Examples of Work

5.5) Why do perpendicular forces not change kinetic energy ??

Recall our definition of work:

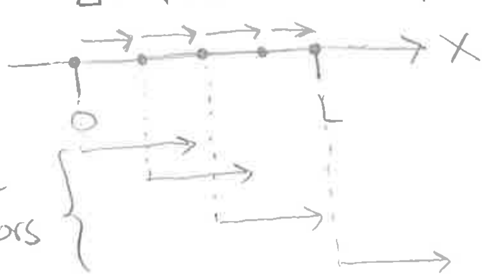
$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Simple examples:

(1) 

Apply constant force along x-direction

$\Delta\vec{r}$ vectors always in +x direction $(\Delta\vec{r} = \Delta x \, \hat{i}_x)$



Force vectors

$\vec{F}$ always in +x direction

$\vec{F} = F\,\hat{i}_x \Rightarrow \Delta\vec{r} \cdot \vec{F} = F\,\Delta x$

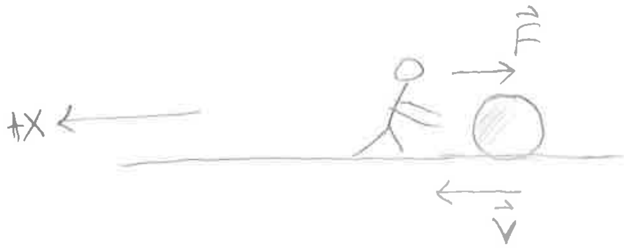
$$W = (\Delta\vec{r}_1)\cdot\vec{F} + (\Delta\vec{r}_2)\cdot\vec{F} + (\Delta\vec{r}_3)\cdot\vec{F} + (\Delta\vec{r}_4)\cdot\vec{F}$$

$$= (\Delta x_1)F + (\Delta x_2)F + (\Delta x_3)F + (\Delta x_4)F$$

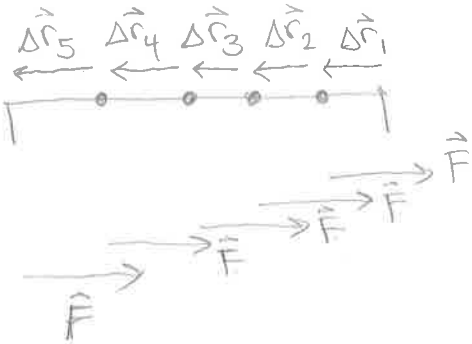
$$= F(\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4)$$

$= FL$ In this case "Work = Force • distance", the algebra definition of work that is sometimes valid.

(2)



Try to stop rolling boulder with a constant force.



Now $\vec{F} \cdot \Delta\vec{r}_i = (-F\hat{i}_x) \cdot (\Delta x\, \hat{i}_x)$

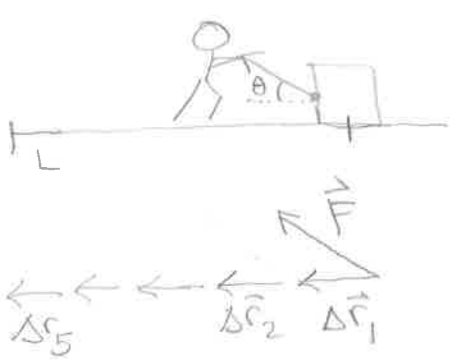
$$\Rightarrow W = \vec{F} \cdot (\Delta\vec{r}_1) + \dots + \vec{F} \cdot (\Delta\vec{r}_5)$$

$$= -F\left[\Delta x_1 + \Delta x_2 + \dots + \Delta x_5\right] = -FL$$

(*) If a force causes an object to speed up (increasing kinetic energy), it does positive work.

(*) If a force causes an object to slow down (decreasing kinetic energy), it does negative work.

(3)



Drag object at angle θ a distance L across floor (frictionless)

Now $\vec{F} \cdot \Delta\vec{r}_i = F(\Delta r_i)\cos\theta$

$$\Rightarrow W = F(\Delta r_1)\cos\theta + F(\Delta r_2)\cos\theta + \dots + F(\Delta r_5)\cos\theta$$

$$= F\cos\theta\left[\Delta r + \dots + \Delta r_5\right] = FL\cos\theta$$

In the three cases above, the force was constant.

(*) We can use the same definition of work for position-dependent forces.

(a) Suppose I get tired dragging blocks around, and the force I can exert decreases with the distance:

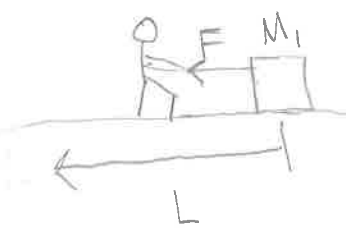
$F(x) = 10 - 5x + x^2$ from $x = 0$ to $x = 2.5$ meters.

How much work do I perform?

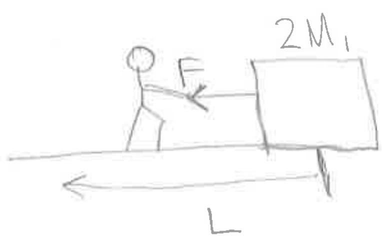
$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_0^{2.5\text{m}} (10 - 5x + x^2)\, dx$$

$$= \left[10x - \frac{5}{2}x^2 + \frac{1}{3}x^3\right]_0^{2.5} = 14.6 \text{ J}$$

(*) Note that work does not explicitly depend on the mass of the object being moved.



$W_1 = FL$



$W_2 = FL$

What does change is the time it takes to move the object a distance L.

"Power" is defined as the work done per unit time.

In the above example, the power is larger in the first case, since the same work is done in a shorter time.

$$P_{average} = \frac{\text{Work}}{\text{time}} \qquad P_{instantaneous} = \frac{dW}{dt}$$

2 Why do perpendicular forces do no work?

Recall $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\vec{v}\cdot\vec{v}$.

Then $\frac{d}{dt}(KE) = \frac{1}{2}m\left(\frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt}\right)$

$$\Rightarrow \frac{d}{dt}(KE) = \frac{1}{2}m\left(\vec{a}\cdot\vec{v} + \vec{v}\cdot\vec{a}\right)$$

$$\frac{d}{dt}(KE) = \frac{1}{2}m\left(2\vec{a}\cdot\vec{v}\right) \qquad \left(\vec{F} = m\vec{a}\right)$$

$$\frac{d}{dt}(KE) = m\left(\frac{\vec{F}}{m}\cdot\vec{v}\right) \qquad \left(\vec{v} = \frac{d\vec{r}}{dt}\right)$$

$$\frac{d}{dt}(KE) = \vec{F}\cdot\frac{d\vec{r}}{dt} \qquad \text{(cancel } dt\text{)}$$

$$d(KE) = \vec{F}\cdot d\vec{r} \qquad \text{(integrate both sides)}$$

$$\int_{\vec{r}_1}^{\vec{r}_2} d(KE) = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

$$\boxed{KE(\vec{r}_2) - KE(\vec{r}_1) = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}}$$

Work-Kinetic Energy theorem follows from Newton's 2nd Law.