# PHYS 206 Lecture 4

## Outline

4.1) Newton's Laws of Motions

4.2) Common types of forces

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## 4.1) Newton's Laws of Motion

* In previous chapters we considered accelerating objects, but apart from gravity we never specified what caused the acceleration.

* Key Point: Acceleration only occurs when there is a net force acting on an object.

  → By "net force" we mean the vector sum of all forces : $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \ldots$

Example: In tug-of-war the rope only accelerates if one team pulls harder, e.g. $|\vec{F}_{left}| > |\vec{F}_{right}|$.

This makes intuitive sense, but how can we express it mathematically?

Precise experiments reveal:

ⓐ If you double the force, you double the acceleration

ⓑ If you apply the same force to an object with twice the mass, the acceleration is half.

In other words

$$\boxed{\vec{F}_{net} = m\vec{a}}$$

Newton's 2nd Law of Motion

Wait, what happened to Newton's 1st Law of Motion?

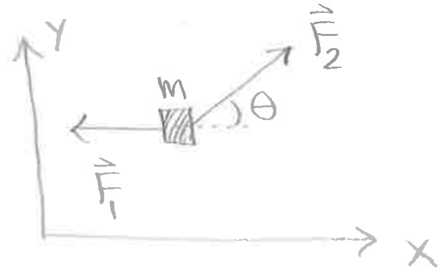
→ Basically it's just the special case of $\vec{F}_{net} = m\vec{a}$ when $\vec{F}_{net} = 0$: "An object in motion stays in motion unless acted upon by an external force."

⊛ Newton's 2nd Law is a vector equation.

→ This means that the equation holds separately for each of the components:

$$\vec{F}_{net} = m\vec{a} \Rightarrow \begin{cases} F_{x,net} = ma_x \\ F_{y,net} = ma_y \\ F_{z,net} = ma_z \end{cases}$$

Example: Consider an object of mass m that moves in the xy plane subject to two constant forces $\vec{F}_1$ and $\vec{F}_2$ shown below. What is the object's acceleration if $F_1$, $F_2$, and $\theta$ are all known?




Solution: From Newton's 2nd Law, $\vec{F}_{net} = m\vec{a}$

$\Rightarrow \vec{F}_1 + \vec{F}_2 = m\vec{a}$. Break up into x and y components separately:

| x motion | y motion |
| --- | --- |
| $F_{x,net} = ma_x$ | $F_{y,net} = ma_y$ |
| $F_2 \cos\theta - F_1 = ma_x$ | $F_2 \sin\theta = ma_y$ |
| $a_x = \dfrac{F_2 \cos\theta - F_1}{m}$ | $a_y = \dfrac{F_2 \sin\theta}{m}$ |

Now that we have $\vec{a} = a_x \hat{i}_x + a_y \hat{i}_y$, we could compute $\vec{v}(t)$ or $\vec{r}(t)$ just like in previous chapters.

(*) Crucial point : As long as the forces depend only on time (or are constant), Newton's 2nd Law will easily produce $\vec{a}(t)$. With a few initial conditions ($v_0$ and $x_0$) we can then uniquely determine the object's complete motion at all times!

For now, most of the forces we consider will be of this form : $F = \text{const.}$ or $F = F(t)$.

For position-dependent forces (like that produced by a spring: $\vec{F} = -k\vec{x}$) we will develop methods in later chapters to determine motion (e.g. work-kinetic energy theorem)

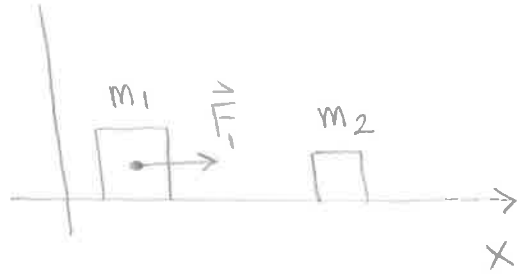
Newton's 3rd Law of Motion : For every force there is an equal and opposite reaction force.

→ In other words, all forces come in pairs but always act on diffent objects and therefore never cancel.

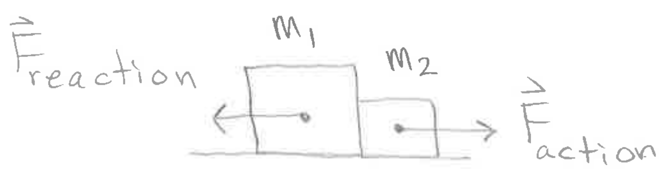
## 4.2) Common types of forces

### 4.2.1) Contact Forces

Example: A block of mass $m_1$ is being pushed by a constant force $\vec{F}_1$ shown below. What happens when block $m_1$ runs into a second block of mass $m_2$?




Solution: Intuitively we expect the acceleration of $m_1$ to decrease, while $m_2$ will start to accelerate. This is due to a pair of equal and opposite action/reaction forces (one acts on $m_1$ and the other acts on $m_2$).

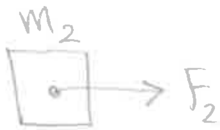



Here $m_1$ pushes $m_2$ to the right, and $m_2$ pushes $m_1$ left.

Since $\vec{F}_{reaction} = -\vec{F}_{action}$, we can say $|\vec{F}_{action}| = F_2$ and $|\vec{F}_{reaction}| = F_2$.

Important**: $\vec{F}_1$ does not act on $m_2$. This is a very common misconception!

Correct "free-body" diagrams for $m_1$ and $m_2$:


$$\vec{F}_{net} = m_1 \vec{a}_1 \qquad\qquad \vec{F}_{net} = m_2 \vec{a}_2$$

$$\boxed{F_1 - F_2 = m_1 a_1} \qquad\qquad \boxed{F_2 = m_2 a_2}$$

But how are $a_1$ and $a_2$ related? They move together and therefore have identical motion (velocities, accelerations):

$\Rightarrow \; a_1 = a_2$

Hence, $F_1 - F_2 = m_1 a$ and $F_2 = m_2 a$ $\left(\begin{array}{l}\text{2 equations} \\ \text{2 unknowns} \rightarrow a, F_2\end{array}\right)$

⊛ The action/reaction forces are typically not known but instead can be computed from Newton's 2nd Law:

(substitute) $F_1 - m_2 a = m_1 a \Rightarrow \boxed{a = \dfrac{F_1}{m_1 + m_2}}$ acceleration of system

(plug back in) $F_2 = m_2 a \Rightarrow \boxed{F_2 = F_1 \left(\dfrac{m_2}{m_1 + m_2}\right)}$ magnitude of action/reaction force pair

Note that the action/reaction force $F_2$ is smaller than $F_1$.

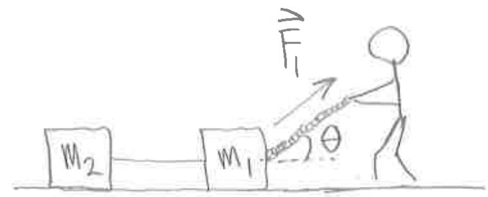
$\rightarrow$ Only "part" of the initial force gets "transferred" to $m_2$.

## 4.2.2) Tension Forces

⊛ Think of tension forces as "pulling" forces

Suppose I pull with a force $\vec{F}_1$ on a rope connected to mass $m_1$ that is connected to a second mass $m_2$ by a thin string.

⊛ Again, the force $\vec{F}_1$ is exerted only on block $m_1$ and not on $m_2$.




The string pulls block $m_2$ to the right, but it also pulls on block 1 to the left (another example of Newton's third Law).

Again there are two objects so we need two free-body force diagrams and two equations:

(Neglect friction)

Box 1: $F_1 \cos\theta - T = m_1 a$

Here $T$ is the tension force and only the horizontal component of $F_1$ causes it to accelerate to the right

Box 2: $T = m_2 a$

Here $T$ has the same magnitude but opposite direction compared to box 1, and again the motions of both boxes are identical so that acceleration $a$ is the same

Similar solution as before :

$$F_1 \cos\theta - m_2 a = m_1 a \quad \Rightarrow \quad a = \frac{F_1 \cos\theta}{m_1 + m_2}$$

Suppose the connecting string will break if it experiences a force greater than $T_c$. What is the maximum force $F_{max}$ we can pull on the rope so that the string doesn't break?

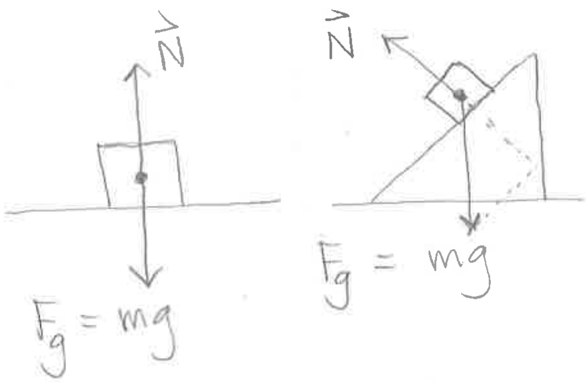
<u>Solution</u>: $T = m_2 a = \dfrac{m_2}{m_1+m_2} F_1 \cos\theta$

$$\Rightarrow F_{max} = \frac{m_1+m_2}{m_2} \frac{T_c}{\cos\theta}$$

Note that if the second mass becomes very small, then it is very difficult to break the string (makes sense intuitively).

## 7.3) Normal forces

⊛ Normal forces are a type of contact force where one object or surface supports another :




⊛ Gravitational force on Earth is always $\boxed{\vec{F}_g = mg\,(-\hat{i}_y)}$

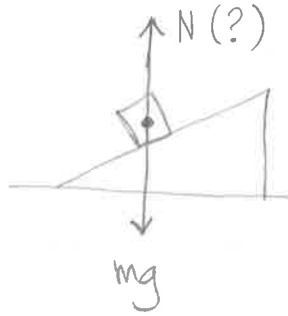
if y direction is upward.

Easily follows from $\vec{F} = m\underbrace{\vec{a}}_{(-g\hat{i}_y)}$

⊛ Normal forces always act perpendicular to the surface of the supporting object

→ In mathematics the "normal vector" to a surface is just the perpendicular vector of length 1.

Question: Why wouldn't the inclined plane example above generate a force straight upward?



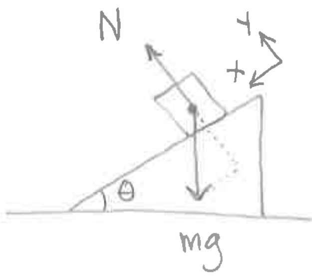

(This doesn't make sense, because part of $mg$ must accelerate block down ramp.)

Answer: Normal forces arise because it is difficult to compress solids, and compression is always perpendicular to a surface.

(In fact, maybe you didn't think it was even possible to compress a solid. Every time you push on a solid object, it does compress ... just by an imperceptible amount.)

Resistance to movement along the direction of a solid is referred to as friction, which we will discuss in more detail at the end of this lecture.

Since solids are very difficult to compress, normal forces can typically grow as large as needed to support an object:




| x direction | y direction |
|---|---|
| $mg\sin\theta$ | $-mg\cos\theta$ |
| | $N$ ← becomes large enough to exactly balance $mg\cos\theta$ |

block only accelerates in x direction, so $a_y = 0$.

$$F_y^{total} = N - mg\cos\theta = 0$$

$$\Rightarrow \boxed{N = mg\cos\theta}$$

← Finding normal forces is very important, because often friction force is proportional to N.

## 7.4) Friction forces

⊛ First of all, friction forces can be tricky!

BE VERY CAREFUL

Sometimes friction acts as we expect:

### Example: Sliding friction

friction force points left


Sliding friction opposes object's velocity

Sliding friction force <u>always</u> given by

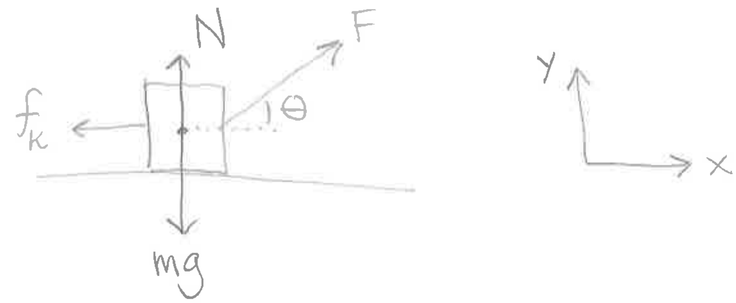
$$f_k = \mu_k N$$

$N$ ← normal force

$\mu_k$ ← coefficient of <u>Kinetic</u> friction (depends on surfaces)

$f_k$ ← "Kinetic"

Example above:




(*) For <u>sliding</u> friction you must <u>always</u> compute normal force:

(y comp): $F\sin\theta + N - mg = 0$ (since no acceleration in y direction)

$$N = mg - F\sin\theta$$

Once we calculated normal force, then friction force is

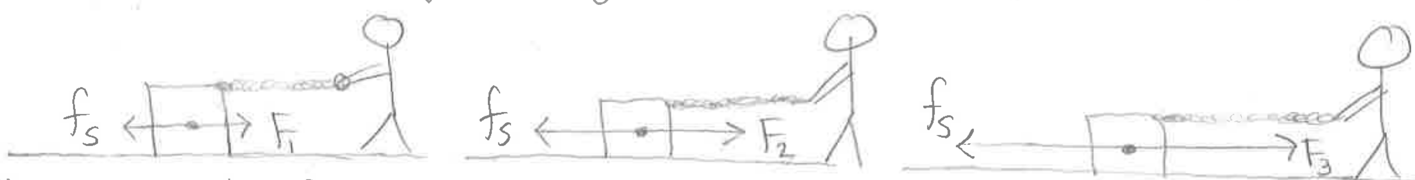
$$f_k = \mu_k N = \mu_k(mg - F\sin\theta)$$

Therefore, $F\cos\theta - \mu_k mg + \mu_k F\sin\theta = ma_x$

$$a_x = \frac{1}{m}\left(F\cos\theta + \mu_k F\sin\theta - \mu_k mg\right)$$

In the above example there was also a friction force acting on the ground from the dragging block ← Newton's 3rd Law.

* Kinetic friction ($f_K$) : Always given by $\boxed{f_K = \mu_K N}$.
* Static friction ($f_s$) : Usually given by $\boxed{\sum F_x = ma_x}$ but just when object begins to move → $\boxed{f_s = \mu_s N}$.




No movement: $f_s = F_1$ No movement: $f_s = F_2$ Begins to move: $f_s = \mu_s N$

* Static friction force grows as the applied force increases (similar to normal force) in order to exactly balance
* Point where sliding begins always given by

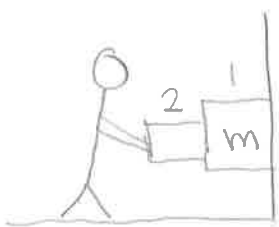
$$f_s = \mu_s N$$

↑ static coefficient of friction

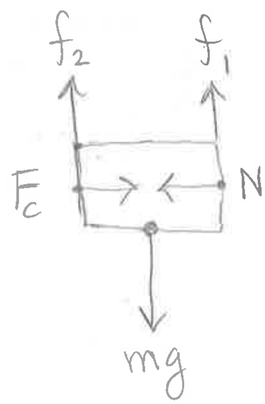
($\mu_s > \mu_K$) always

Example: Suppose you hold one box against a wall by pushing a second box against it with force $F(t) = F_0 - \alpha t$. If the first box has mass $m$ and the coefficients of static friction on its two sides are both equal to $\mu$, at what time will it begin to fall?

Solution: Friction is the only force keeping the box from falling.

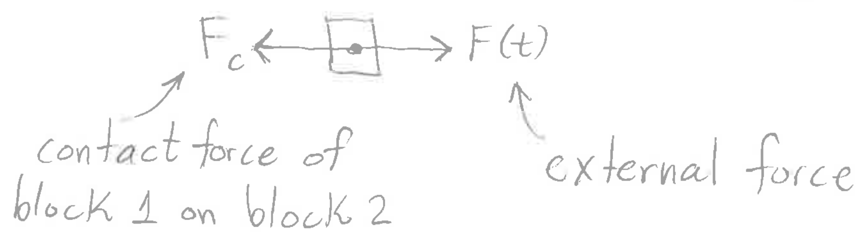







(block 1)

(block 2)




$$F(t) - F_c = 0 \Rightarrow F_c = F(t)$$ ← since block 2 does not move

Block 1: $f_1 = \mu N$ and $f_2 = \mu F_c$ when block first begins to slide. But $N = F_c$ since block 1 does not move in $x$ direction.

Therefore, $mg - \mu F(t) - \mu F(t) = 0$

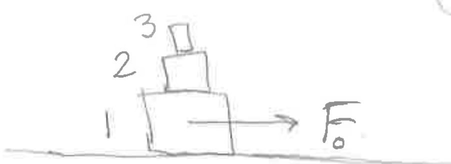
$$\Rightarrow mg - 2\mu (F_0 - \alpha t) = 0$$

$$\Rightarrow \frac{mg}{2\mu} = F_0 - \alpha t$$

$$\Rightarrow t = \frac{1}{\alpha}\left[F_0 - \frac{mg}{2\mu}\right]$$

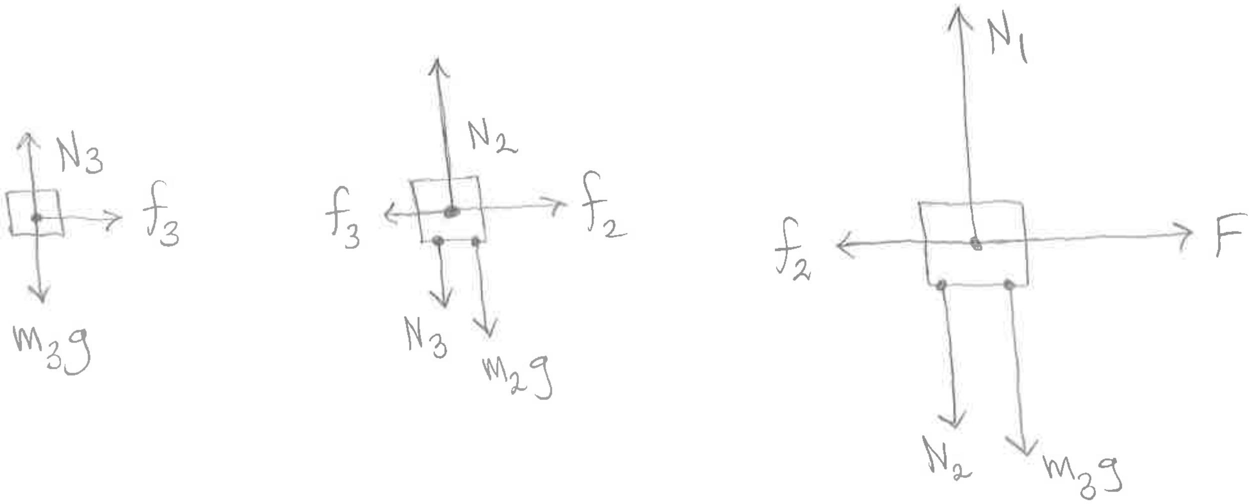
(*) Friction does not always act against direction of motion.

Example: Suppose 3 blocks with friction between them accelerate together due to external force $F_0$. But




suppose there is no friction between block 1 and ground.

(*) Blocks 2 and 3 move to the right due entirely to friction (otherwise they would just slide off)




Block 3 : (y dir) $N_3 - m_3 g = 0$

$$N_3 = m_3 g$$

(x dir) $f_3 = m_3 a_x$ ($f_3 \neq \mu N_3$ since not sliding)

Block 2 : (y dir) $N_2 - m_3 g - m_2 g = 0$

$$N_2 = (m_2 + m_3) g$$

(x dir) $f_2 - f_3 = m_2 a_x$

Block 1 : (y dir) $N_1 - (m_2 + m_3) g - m_1 g = 0$

$$N_1 = (m_1 + m_2 + m_3) g$$

(x dir) $F_0 - f_2 = m_1 a_x$

3 x-direction equations and unknowns $(a_x, f_2, f_3)$.

$$\Rightarrow \quad f_2 - f_3 = m_2 a_x$$

$$f_2 - m_3 a_x = m_2 a_x$$

$$f_2 = (m_2 + m_3) a_x$$

$$\Rightarrow \quad F_0 - (m_2 + m_3) a_x = m_1 a_x$$

$$\boxed{a_x = \frac{F_0}{m_1 + m_2 + m_3}}$$

Makes sense since $F_0$ must accelerate the combined mass $(m_1 + m_2 + m_3)$ together.

$$\Downarrow \qquad\qquad \Downarrow$$

$$\boxed{f_2 = \frac{m_2 + m_3}{m_1 + m_2 + m_3} F_0} \qquad \boxed{f_3 = \frac{m_3}{m_1 + m_2 + m_3} F_0}$$

(*) Note that we never used $f_K = \mu N$ to compute any of these friction forces!

When would we use $f_\mu = \mu N$? We could have asked "At what value of $F_0$ will block $m_3$ start to slide off of $m_2$?" Suppose the coefficient of "static" friction is $\mu_s$.

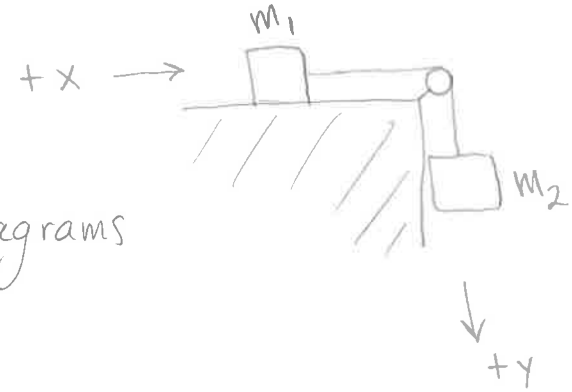
Answer: For sliding friction use $f_3 = \mu_s N = \mu_s m_3 g$

$$\Rightarrow \quad \mu_s m_3 g = \frac{m_3}{m_1 + m_2 + m_3} F_0 \quad \Rightarrow \quad \boxed{F_0 = \mu_s (m_1 + m_2 + m_3) g}$$

lighter box slides off earlier

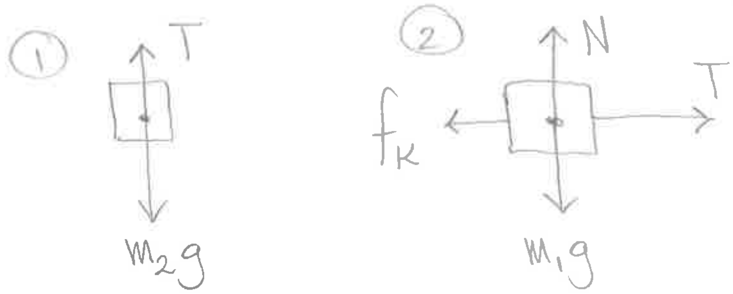
Example : Pullys plus friction

Suppose two blocks are connected by a thin wire that can slide without friction across a massless pulley. If the two blocks have masses $m_1$ and $m_2$ and the coefficient of friction between $m_1$ and the ground is $\mu$, what will be the acceleration of the system when it begins to move due to gravity?




Solution

Two objects → two free-body diagrams




Block 2) $m_2 g - T = m_2 a$ (Note that I have chosen the "+" direction to be along acceleration. Very important! Otherwise, the two accelerations $a_1$ and $a_2$ could have different signs.)

Block 1) $x: \; T - f_K = m_1 a \;\Rightarrow\; T - \mu N = m_1 a$

$y: \; N - m_1 g = 0 \;\Rightarrow\; N = m_1 g$

$$\Rightarrow \quad T - \mu m_1 g = m_1 a$$

$$\Rightarrow \quad T = m_1(a + \mu g)$$

Plug back into $m_2$ equation:

$$m_2 g - m_1(a + \mu g) = m_2 a$$

$$m_2 g - m_1 \mu g = (m_1 + m_2) a$$

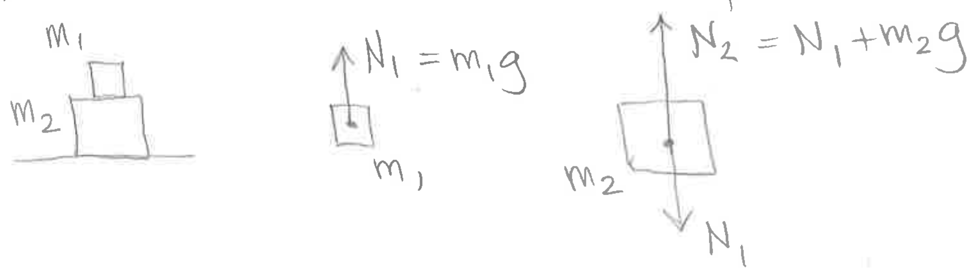
$$\boxed{a = \frac{m_2 - \mu m_1}{m_1 + m_2} g}$$

## Recap:

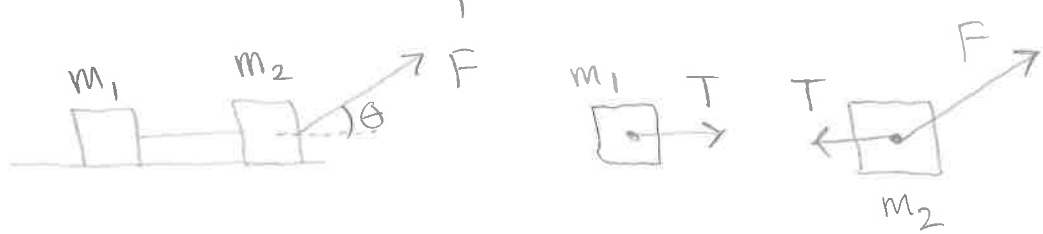
⊛ 4 types of forces

(1) Gravity on Earth surface: $F = mg$ (downward)

(2) Contact (normal force): always perpendicular to surface, grows to exact value needed to produce some given motion (no explicit formula), always part of an action-reaction pair:




(3) Tension: always parallel to wire direction, grows to exact value needed to produce some given motion (no explicit formula), always part of an action-reaction pair:




(4) Friction: "Sliding friction" → $f_K = \mu_K N$

"Static friction" → grows to exact value needed to produce some given motion.

PHYS 206 Lecture 4 

## ⊛ Applying Newton's 2nd Law

Step 1) Draw a separate free-body diagram for each object. Only forces acting on an object should be drawn.

Step 2) Break up forces into components along the axes of some convenient coordinate system.

Step 3) Write down a separate $F^x_{tot} = m_1 a^x_1$, $F^y_{tot} = m_1 a^y_1$ set of equations for mass $m_1$, $m_2$, etc.

Step 4) Identify equal/opposite force pairs that have the same magnitude.

Step 5) Identify objects having the same acceleration