PHYS 206 Lecture 2

2.1) Vectors

2.2) Vector algebra

2.1) Vectors

* Definition: a "vector" is a quantity that has a magnitude and direction

First, some quantities that are not vectors:

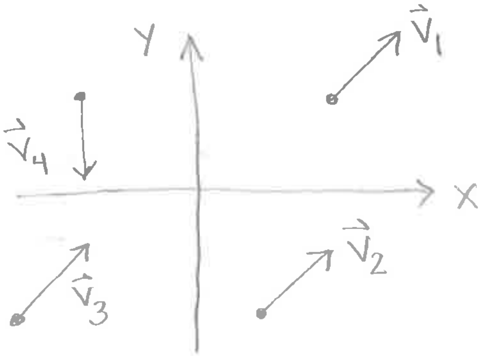
mass, volume, temperature, energy $\{m, V, T, E\}$

Next, some quantities that are vectors:

vector symbol

position, velocity, acceleration, force $\{\vec{r}, \vec{v}, \vec{a}, \vec{F}\}$

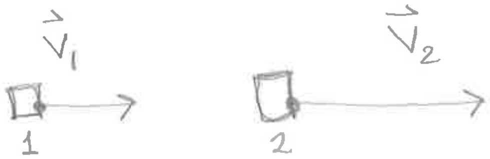
* Typically, vectors are independent of position (except, of course the position vector):



4 objects, each with a velocity vector shown.

$$\vec{v}_1 = \vec{v}_2 = \vec{v}_3 \neq \vec{v}_4$$

* We denote the magnitude of the vector by its length and the direction by the directed arrow

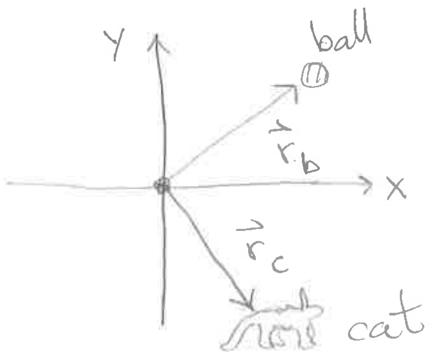


$|\vec{v}_1| < |\vec{v}_2|$

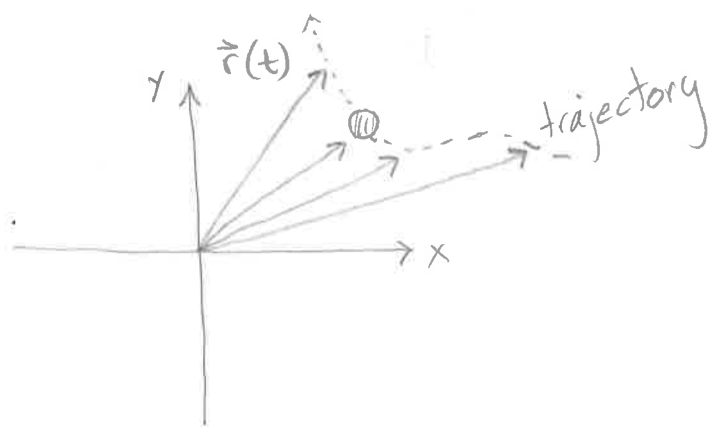
Two objects travel in same direction, but object 2 travels faster.

Magnitude denoted with absolute value symbol or just v_1 and v_2 without the vector symbol.

* Position vector $\vec{r}$ is special. It <u>always</u> points from the origin to the location of the object:



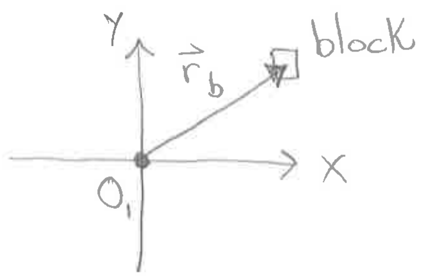
[Position vectors of ball and cat.]

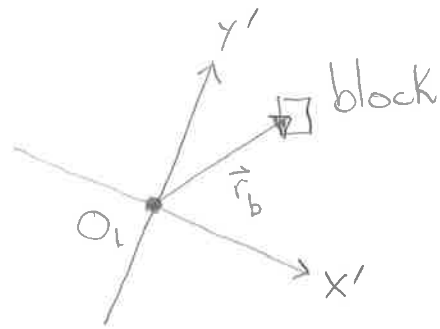


[Position vector $\vec{r}(t)$ as a function of time.]

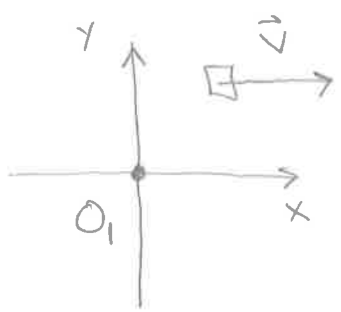
* Even though the object can be in 2 or 3 dimensional space, it is common to also use $\vec{x}$ for position vector.

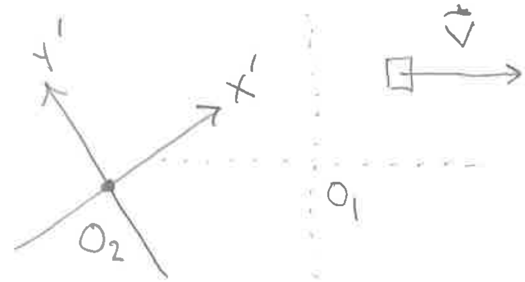
(*) The position vector depends on the choice of origin, but not on the choice of x and y directions.





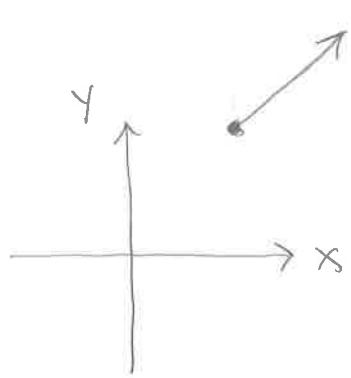
(*) Most other vectors remain unchanged when the coordinate system is shifted or rotated:



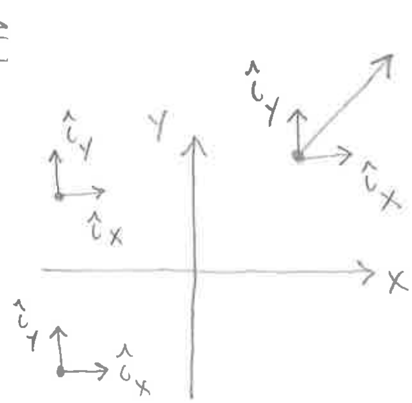


(*) Never confuse a vector (which is defined by its magnitude and direction) by its components in a given coordinate system.

Definition : Given a set of basis vectors, defined by the choice of coordinate system, a vector can be resolved into its components along those basis vectors as follows:

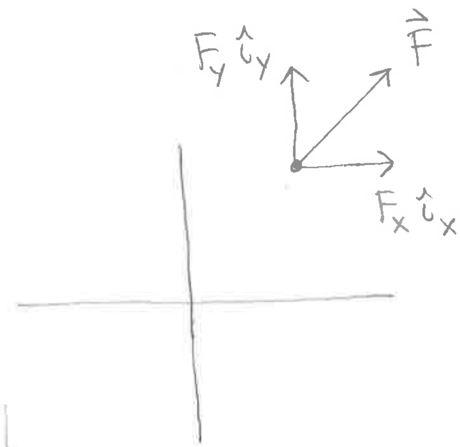


Suppose we have a force vector $\vec{F}$ in a given coordinate system.



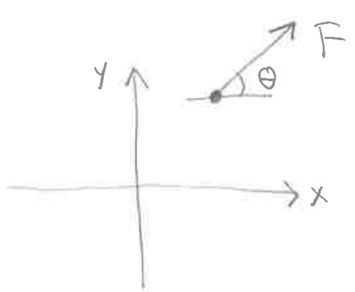
$\hat{\imath}_x \equiv$ vector of magnitude 1 along x-direction

$\hat{\imath}_y \equiv$ vector of magnitude 1 along y-direction



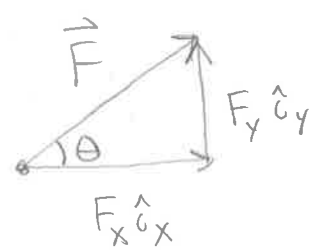
F_x and F_y are just numbers. Only $F_x \hat{\imath}_x$ and $F_y \hat{\imath}_y$ are vectors.

(*) If we know the angle that a vector makes with respect to either the x or y axis, then we can uniquely calculate the components of the vector:



$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$



PHYS 206 Lecture 2

2.2) Vector Algebra

* We can

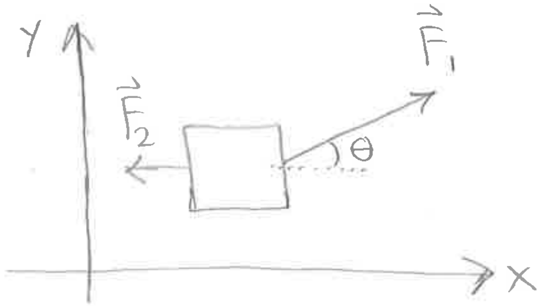
(a) Add/Subtract two vectors

(b) Multiply a vector by a number

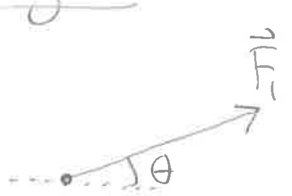
(c) Multiply two vectors (either "dot product" that produces a number or "cross product" that produces a vector)... later in semester

(a) Add/Subtract vectors is easy → just add/subtract their x and y components separately.

Example: Given the two force vectors $\vec{F}_1$ and $\vec{F}_2$ acting on some object, find the total force. The magnitudes F_1 and F_2 are known.



Solution: Always, always, always break up forces into their x and y components, Keeping track of all + and − signs.



$$\Rightarrow \quad F_{1x} = F_1 \cos\theta$$
$$F_{1y} = F_1 \sin\theta$$

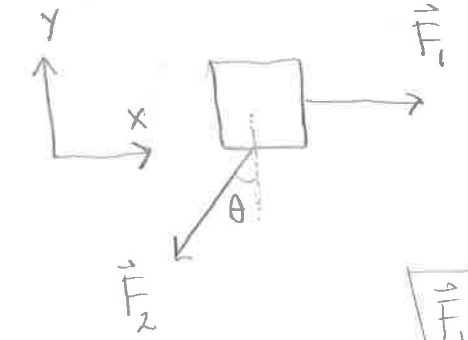
$$\Rightarrow \quad F_{2x} = -F_2 \quad \text{(note minus sign!)}$$
$$F_{2y} = 0$$

$$\boxed{\vec{F}_{total} = (F_1 \cos\theta - F_2)\,\hat{i}_x + F_1 \sin\theta\,\hat{i}_y}$$

Note that $\vec{F}_2$ makes an angle $\theta_2 = 180°$ with respect to the $+x$ axis. Therefore we could have used

$$F_{2x} = F_2 \cos(180°) = F_2(-1) = -F_2$$
$$F_{2y} = F_2 \sin(180°) = F_2(0) = 0$$

(*) Important: Do not blindly assume x component will always have a $\cos\theta$ and y component will have a $\sin\theta$:



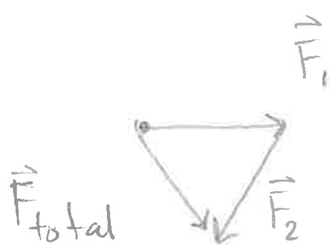
$F_{1x} = F_1 \qquad F_{2x} = -F_2 \sin\theta$

$F_{1y} = 0 \qquad F_{2y} = -F_2 \cos\theta$

Note the two minus signs!

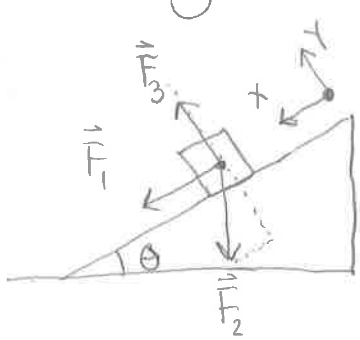
$$\boxed{\vec{F}_{total} = (F_1 - F_2 \sin\theta)\,\hat{\imath}_x - F_2 \cos\theta\,\hat{\imath}_y}$$

⊛ One can add vectors using the "tip-to-tail" method, but usually it is simpler to just add components



(Example)

⊛ Adding vectors along rotated axes:



$F_{1x} = F_1 \qquad F_{2x} = F_2 \sin\theta$

$F_{1y} = 0 \qquad F_{2y} = -F_2 \cos\theta$

$F_{3x} = 0$

$F_{3y} = F_3$

$$\boxed{\vec{F}_{total} = (F_1 + F_2 \sin\theta)\,\hat{\imath}_x + (F_3 - F_2 \cos\theta)\,\hat{\imath}_y}$$

⊛ Multiplying vectors by numbers ("scalars"):

(a) Direction stays same

(b) Magnitude of vector is multiplied by the scalar

<u>Example:</u>