Electricity and Magnetism PHYS 208

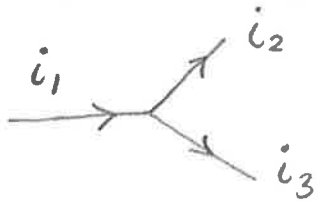
11. Time-independent circuits

11.1 Kirchhoff's Laws
11.2 Resistors in parallel and series
11.3 Power in circuits

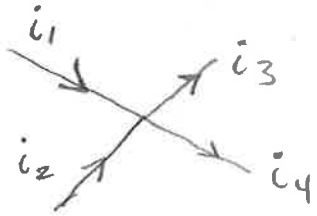
11.1 Kirchhoff's Laws

⊛ There are two essential rules for analyzing circuits that generally go by the name "Kirchhoff's Laws"

① The current that enters a junction must equal the current leaving the junction



$$i_1 = i_2 + i_3$$



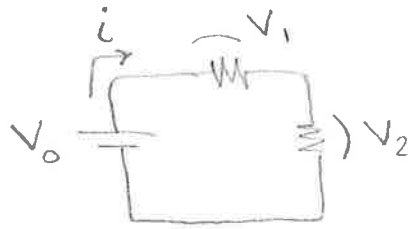
$$i_1 + i_2 = i_3 + i_4$$

⊛ Note that this is really just a statement of current conservation:

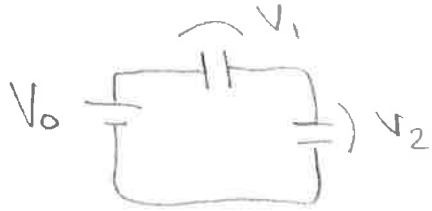
$$\oint \vec{j} \cdot d\vec{S} = 0$$

The current entering a closed surface must equal current exiting

(2) $\Delta V = 0$ around any closed loop



$$V_0 - V_1 - V_2 = 0$$



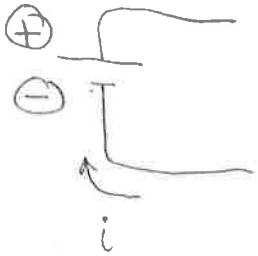
$$V_0 - V_1 - V_2 = 0$$

(*) Voltage "drops" occur across <u>resistors</u> (in the direction of current flow) and across <u>capacitors</u> (in the direction of ⊕ charge to ⊖ charge)

$$\Delta V_r = IR \quad \text{(resistor)}$$

$$\Delta V_c = \frac{Q}{C} \quad \text{(capacitor)}$$

(*) Voltage "gains" occur across batteries (in the direction of negative terminal to positive terminal)

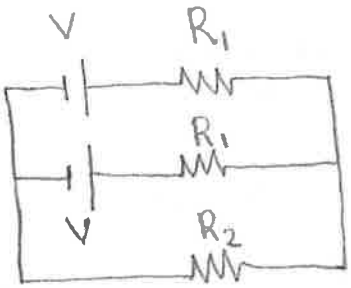


⊛ Note that this law is really just a statement that the electric force is conservative

$$V(a) - V(a) = -\int_a^a \vec{E} \cdot d\vec{l} = 0$$

⊛ Kirchhoff's Laws are nothing new, but they are very convenient for analyzing complicated circuits

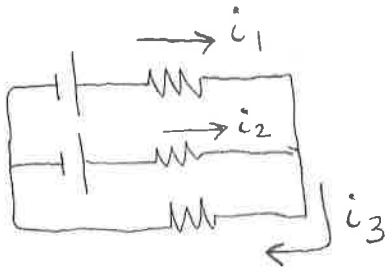
Example:



Solution:

Step 1: Always start by labeling all currents $i_1, i_2, i_3, \ldots$ and their directions.

Label however you want.



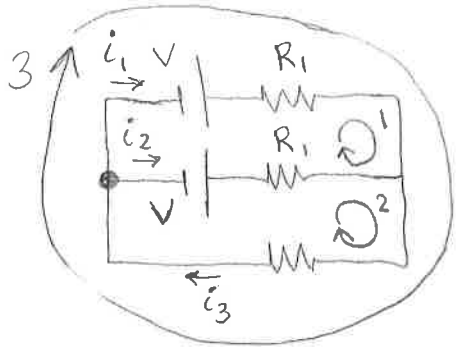
Step 2: Use the current conservation rule. In this case

$$i_1 + i_2 = i_3$$

to obtain one equation relating the unknown currents.

Note that since there are 3 variables (i_1, i_2, i_3), we will need two more equations.

Step 3: Get remaining equations from the loop rule:



For this circuit there are 3 possibilities, but we only need 2!

You can "start" the loop anywhere. Let's choose the solid point.

(1) $V - i_1 R_1 + i_2 R_1 - V = 0$

because we're going from ⊕ to ⊖ terminal

⊕ because loop direction is opposite i_2 current direction we chose

(2) $V - i_2 R_1 - i_3 R_2 = 0$

Now the loop direction is in the same direction as our chosen i_2 and i_3.

(3) $V - i_1 R_1 - i_3 R_2 = 0$

Only two of these are "independent".

From (1) we find $i_1 = i_2$ (and then clearly (2) is same as (3))

From (2) we find

$$V - i_2 R_1 - (i_1 + i_2) R_2 = 0$$

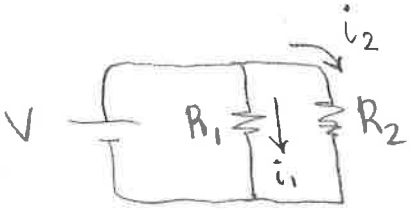
$$V - i_2 R_1 - 2 i_2 R_2 = 0$$

$$V = i_2 (R_1 + 2R_2) \Rightarrow i_2 = \frac{V}{R_1 + 2R_2} = i_1$$

Finally, $i_3 = 2 i_2 = \dfrac{2V}{R_1 + 2R_2}$

11.2) Resistors in parallel and series

(*) Since resistance $R = \rho \frac{L}{A}$ decreases as the conducting area increases, adding resistors in parallel decreases the effective combined resistance



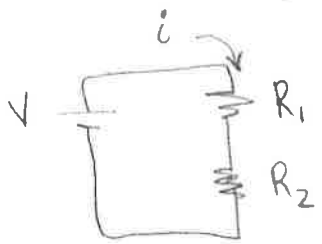
$$V = i_1 R_1 \qquad V = i_2 R_2$$

$$V \equiv (i_1 + i_2) R_{eq}$$

$$= \left(\frac{V}{R_1} + \frac{V}{R_2}\right) R_{eq}$$

$$\Rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

(*) Since resistance increases with length ($R = \rho \frac{L}{A}$), adding resistors in series will increase the combined effective resistance



$$V = iR_1 + iR_2$$

$$V \equiv iR_{eq}$$

$$\Rightarrow \boxed{R_{eq} = R_1 + R_2}$$

Focus Question: Exam 2 (2012) Problem 1

Both parts

11.3) Power in circuits

Recall that mobile charges in a circuit travel with a roughly constant drift velocity $\vec{u}$ in identical segments of wire.

(*) But the $\vec{E}$ field driving the charges does <u>work</u>

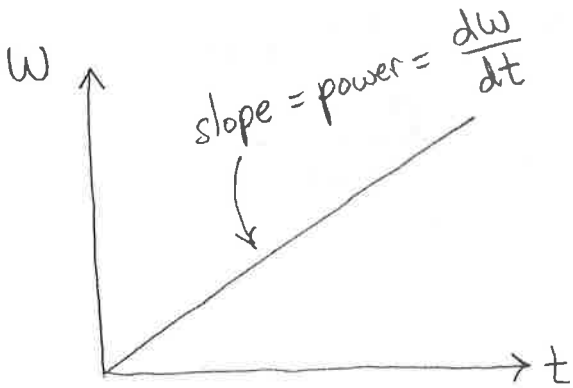
$$W = \int \vec{F} \cdot d\vec{l} = \int q\vec{E} \cdot d\vec{l} = q\Delta V$$

total voltage change across part or all of circuit

Since drift velocity $\vec{u}$ is the same coming out of the ⊕ terminal and going into the ⊖ terminal, the energy $q\Delta V$ does <u>not</u> go into increasing the overall kinetic energy.

<u>Question</u>: Where does the energy $q\Delta V$ go?

<u>Answer</u>: It must be <u>lost</u> by the charges and transferred to the rest of the metal in the form of heat or light



Once circuit reaches a steady state, the energy output per second is constant:

$$P = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = \frac{d}{dt}(q\Delta V) = \frac{dq}{dt}\Delta V = I\Delta V$$

$$\boxed{P = I\Delta V}$$

Question: Does a battery supply a constant power that is independent of the circuit elements we attach to it?

Answer:

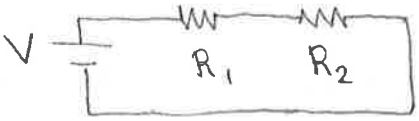
No. A battery supplies a constant voltage ΔV, but the current I will depend on what we connect to the battery.

(*) Note that there are several equivalent forms of the power equation:

$$P = I\Delta V = I(IR) = \boxed{I^2 R}$$

$$= \left(\frac{\Delta V}{R}\right)\Delta V = \boxed{\frac{\Delta V^2}{R}}$$

The power law holds for individual components of a circuit



$$V = I(R_1 + R_2) \Rightarrow I = \frac{V}{R_1 + R_2}$$

$$P_1 = I^2 R_1 = \frac{V^2}{(R_1+R_2)^2} R_1 .$$

Is this energy output good or bad?

Good: light bulbs, toasters, ..

Bad: overheating laptop, hot light bulbs, ...