10.1) Conservation of Angular Momentum

10.2) Applications

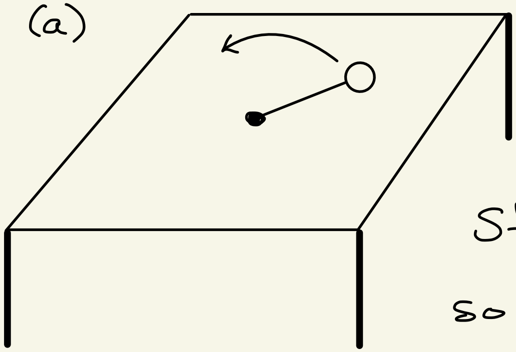
10.1) Conservation of Angular Momentum

- We already showed last lecture that a freely moving object has a constant angular momentum.
- More generally, angular momentum will be conserved when there are no <u>external torques</u> acting on the system:

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt} \quad \left(\text{like } \vec{F} = \frac{d\vec{p}}{dt}\right)$$

<u>Examples where $\vec{\tau}_{ext} = 0$:</u>

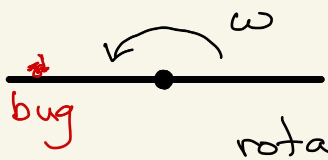
(a)



Ball held in circular motion on frictionless table by string.

String always acts radially, so $\vec{\tau}_{ext} = \vec{r} \times \vec{F} = 0$.

(b)



Solid body freely rotating on frictionless axel.

Even if bug crawls on it, there is no external torque acting on the combined rod + bug system.

(c)

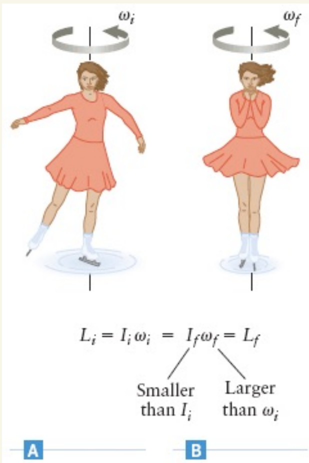
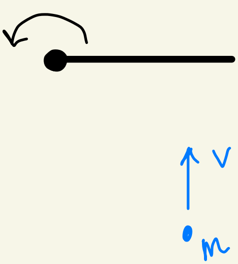


Figure skater rotating on frictionless ice. Bringing arms in close to body reduces I and so ω must increase to keep $L = I\omega$ constant.

(d)

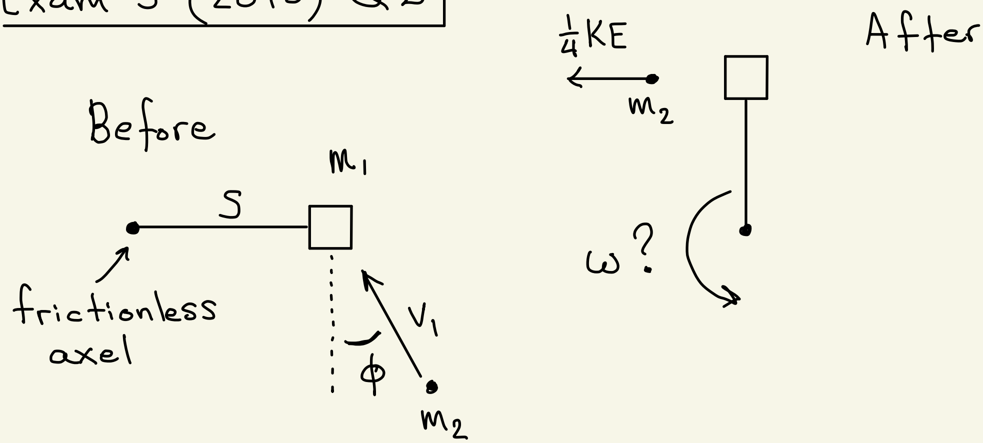


Angular momentum conserved in collisions because no external torques act during that time.

10.2) Applications

Let's look at some sample problems that put everything together.

Exam 3 (2010) Q2

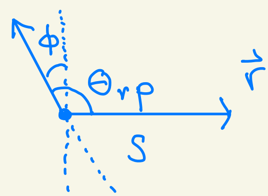


⊛ If you see a collision question, first think "angular momentum conservation".

Angular momentum before collision:

Block at rest $\Rightarrow 0$

When bullet makes impact



$$\vec{L}_b = \vec{r} \times \vec{p} = rp\sin\theta_{rp} \odot$$

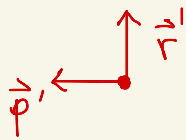
$$= rp\sin\left(\frac{\pi}{2}+\phi\right) \odot = s m_2 v_1 \cos\phi \odot$$

Angular momentum after collision:

Block: $\vec{L} = I\omega \odot = (m_1 s^2)\omega \odot$

Bullet: $KE_f = \frac{1}{2}mv_f^2 = \frac{1}{4}KE_f = \frac{1}{4}\left(\frac{1}{2}m_2 v_1^2\right)$

$$\Rightarrow v_f = \frac{1}{2}v_1$$



$$\vec{L}\,'_b = \vec{r}\,' \times \vec{p}\,' = s m_2 v_f \underbrace{(\sin 90^\circ)}_{=1} \odot$$

$$= \frac{1}{2} s m_2 v_1 \odot$$

$$L_f = m_1 s^2 \omega + \frac{1}{2} s m_2 v_1$$

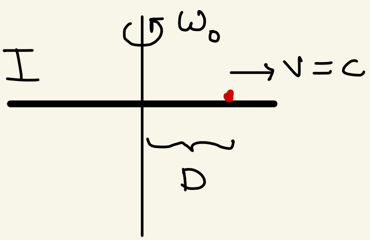
$$L_i = L_f$$

$$s m_2 v_1 \cos\phi = m_1 s^2 \omega + \frac{1}{2} s m_2 v_1$$

$$\Rightarrow \omega = \frac{1}{m_1 s^2}\left(s m_2 v_1 \cos\phi - \frac{1}{2} s m_2 v_1\right)$$

$$\omega = \frac{m_2}{m_1 s} v_1 \left(\cos\phi - \frac{1}{2}\right)$$

Exam 3 (2011) Q2



(a) What is bug's velocity

(b) Torque exerted by rod on the bug?

To find velocity, we need to find how fast rod rotates $\longrightarrow$ get using angular momentum conservation:

$$L_i = I_i \omega_i = (I + mD^2)\omega_0$$

$$L_f = I_f \omega_f = (I + m\underbrace{(D+ct)}_{r(t)}{}^2)\,\omega_f$$

$$\Rightarrow \omega_f = \frac{I + mD^2}{I + m(D+ct)^2}\,\omega_0$$

Then $\vec{v}_b = \frac{dr}{dt}\hat{i}_r + r\omega\hat{i}_\theta$

$$= c\,\hat{i}_r + (D+ct)\,\omega_f\,\hat{i}_\theta$$

How to find torque? Need to first find the force:

$$\vec{F} = m\vec{a} = m\left[\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{i}_r + \left(r\alpha + 2\omega\frac{dr}{dt}\right)\hat{i}_\theta\right]$$

Then $\vec{\tau} = \vec{r} \times \vec{F} = r\hat{i}_r \times \left[\cancel{ma_r \hat{i}_r}^{\,0} + ma_\theta \hat{i}_\theta \right]$

$\hat{i}_r \times \hat{i}_r = 0$

$$= r m a_\theta = rm\left(r\alpha + 2\omega \frac{dr}{dt} \right)$$

$$\left[\begin{array}{l} r(t) = D + ct \rightarrow \frac{dr}{dt} = c \\ \omega(t) = \frac{D^2}{(D+ct)^2}\omega_0 \rightarrow \alpha = \frac{-2D^2(c)}{(D+ct)^3} \end{array} \right.$$

$$\Rightarrow r\alpha + 2\omega\frac{dr}{dt} = (D+ct)\left[\frac{-2cD^2}{(D+ct)^3} \right] + 2\frac{D^2}{(D+ct)^2}c$$

$$= 0$$

$$\Rightarrow \vec{\tau} = 0$$