1. Electric Charge and Coulomb's Law

1.1) Electric charges, conductors, and insulators

1.2) Electric forces

1.3) Review of Key concepts from Mechanics

1.1) Electric charges, conductors, and insulators

* Physics is the study of matter and its interactions.

Everyday matter consists entirely of just three particles: protons, neutrons, and electrons

These particles interact through 4 fundamental forces:

(a) Gravity (purely attractive interaction between objects with mass)

(b) Electromagnetic

(c) Strong force (acts only among protons and neutrons to tightly bind them in atomic nuclei)

(d) Weak force (acts among protons, neutrons, and electrons but only in subtle and generally rare events)

* The phenomena associated with the electromagnetic force are so varied and rich that we need an <u>entire</u> <u>course</u>!

However, there is one key concept that underlies all of these phenomena: <u>electric</u> <u>charge</u>

* All charges can be classified as

Positive	Negative	Neutral
protons	electrons	neutrons
⋮	⋮	⋮

} There are other particles in nature but not relevant for this course.

<u>Chapters 1-6</u>: Charges at rest ("electrostatics").
Electric forces, electric fields, electric potential.

<u>Chapters 7-12</u>: Charges in motion. Electric currents and circuits, origin of magnetic forces and fields.

<u>Chapter 13</u>: Accelerating charges. Sources of electromagnetic waves (light, radio waves, microwaves, X-rays, ...).

Question : Which of the two common charged particles (protons, electrons) is more mobile ?

Answer : Electrons (they are much less massive)

	Protons	Electrons	Ratios
Mass :	1.7×10^{-27} Kg	9.1×10^{-31} Kg	$\frac{m_p}{m_e} \approx 2000$
Charge :	1.6×10^{-19} C	-1.6×10^{-19} C	$\frac{q_p}{q_e} = -1$

SI unit of charge is the Coulomb

charge commonly denoted by q or Q.

* Electrons are so mobile, they can be transferred or shared between atoms (resulting in molecular bonds)

* Electron mobility varies greatly from one material to another

(1) Conductors : Materials containing highly mobile charges (think of electrons as forming a fluid that can flow).

(2) Insulators : Materials whose charges cannot flow under normal circumstances. However, insulators can be charged, those charges just can't move freely.

Question: Is air a conductor or insulator?

Answer: Normally it is an insulator. For example, if there is a break in a conducting wire, electrons cannot flow across the break.

But under extreme conditions (very high temperatures $T > 10,000\ K$ or systems with large charge imbalances such as thunderstorms) air can be a conductor

1.2) Electric Forces

* All electric forces can be described by a universal law governing the interaction between two particles with charges q_1 and q_2:

Coulomb's Law

$$\boxed{F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}}$$

(magnitude of electric force)

Key Points

1. Soon we will show that $\frac{1}{4\pi\varepsilon_0}$ has a geometric origin (note that $4\pi r^2$ in the denominator is surface area of a sphere). Therefore physicists prefer $\frac{1}{4\pi\varepsilon_0}$ rather than Coulomb's constant $K \equiv \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2/C^2$.

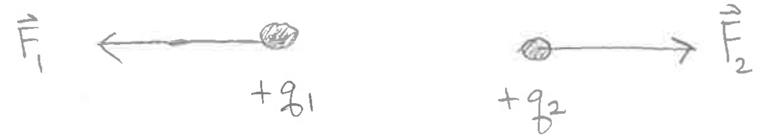
(2.) Direction of electric force

Opposite charges each experience an attractive force, while like charges each experience a repulsive force.

A neutral object generally won't experience an electric force, but in reality it can be polarized if it is composed of equal + and − charges (leads to small net attraction).

(3) From Newton's 3rd Law, the charges q_1 and q_2 experience equal and opposite forces

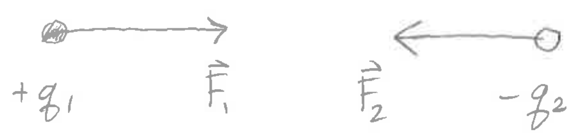
Examples:



$|\vec{F}_1| = |\vec{F}_2|$

(equivalent notation)

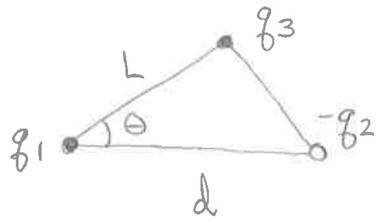
$F_1 = F_2$



$F_1 = F_2 = 0$

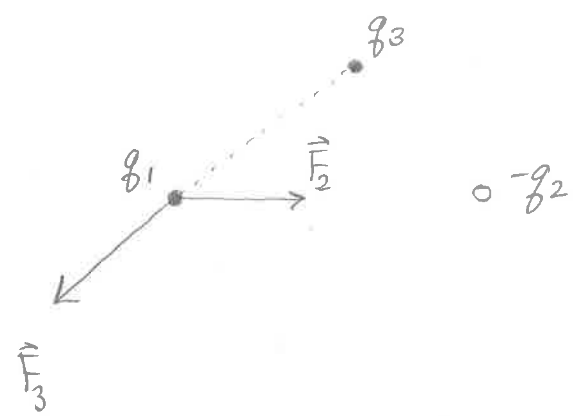
(4) If there are more than 2 charges present, compute pairwise forces separately and add as vectors

Key Example The charges q_1, $-q_2$, and q_3 are fixed at the locations shown. What is the net force on q_1?



Solution: Follow the 4-step procedure

① Draw the individual force vectors (including direction) on charge q_1:



⊛ Note: we have drawn $\vec{F}_3$ away from q_3 (repulsive force) and $\vec{F}_2$ toward q_2 (attractive force). Don't worry about their magnitudes for this step.

⊛ Note: charge q_1 also exerts forces on q_3 and q_2, but do not draw them... they are not our problem.

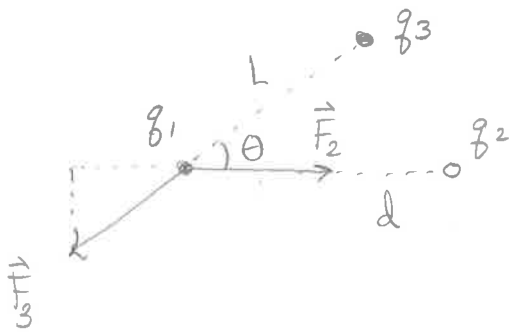
② Calculate the magnitudes of all drawn forces (Coulomb's Law):

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$$

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{L^2}$$

Crucial point is that we do not include signs of charges because we have already accounted for direction in step ①.

③ Break up forces into their x and y components:



$$F_{2x} = +F_2 = +\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$$

$$F_{2y} = 0$$

$\vec{F}_2$ points directly right, so F_{2x} gets positive sign and $F_{2y} = 0$.

$$F_{3x} = -F_3\cos\theta = -\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{L^2}\right)(\cos\theta)$$

$$F_{3y} = -F_3\sin\theta = -\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{L^2}\right)(\sin\theta)$$

$\vec{F}_3$ points left and downward, so both F_{3x} and F_{3y} get negative signs.

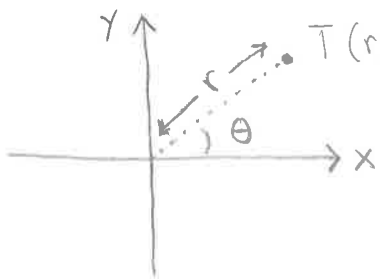
④ Add x and y components <u>separately</u> (this is essential when adding a vector quantity like force):

$$\boxed{\vec{F}_{total} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{d^2} - \frac{q_1 q_3}{L^2}\cos\theta\right)\hat{i}_x - \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{L^2}\sin\theta\right)\hat{i}_y}$$

1.3) Review of Key Concepts from Mechanics

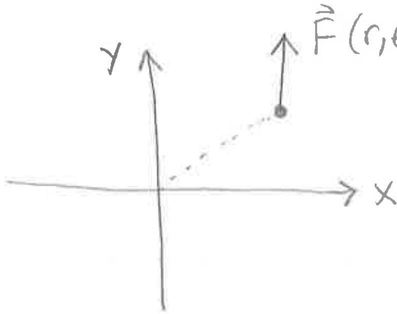
1.3.a) Polar Coordinates

- Scalar functions $f(r,\theta)$ take a numerical value at each location (r,θ). Examples: temperature, potential energy, ...



$T(r,\theta)$ = temperature at (r,θ)

- Vector functions $\vec{v}(r,\theta)$ take a vector value at each location (r,θ). Examples: force, velocity, angular momentum, ...



$\vec{F}(r,\theta)$ = force on object at (r,θ)

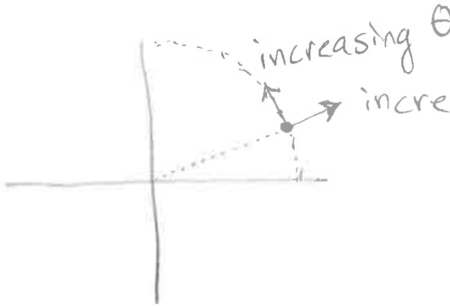
Important Point: to describe vectors in polar coordinates, we need to understand "basis vectors" in polar coordinates. The basis vectors $\hat{i}_r$ and $\hat{i}_\theta$ depend on θ but not on r.

How do we define basis vectors?

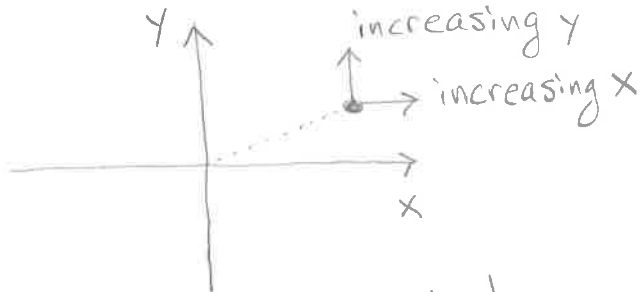
(1) Start at an arbitrary point $\vec{r}_1$

(2) Increase r while keeping θ fixed $\rightarrow$ defines $\hat{i}_r$

(3) Increase θ while keeping r fixed $\rightarrow$ defines $\hat{i}_\theta$

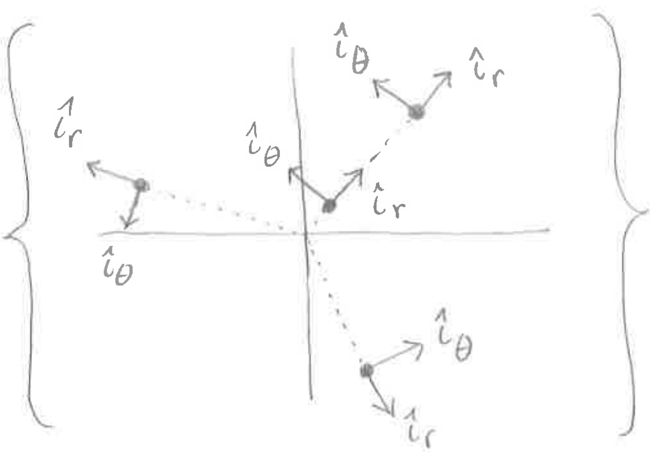


Polar Basis Vectors



Cartesian Basis Vectors

Consider basis vectors at 4 different positions



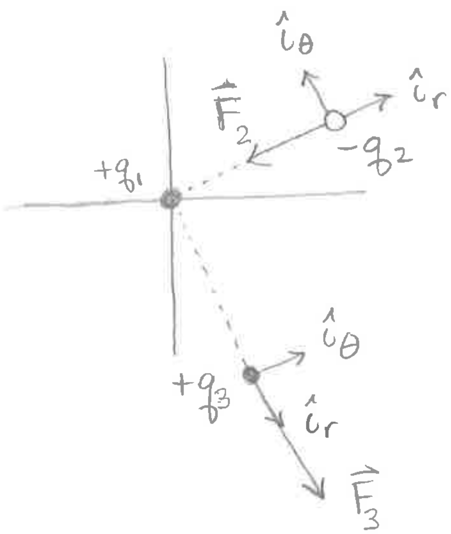
Polar Basis Vectors depend on θ but not on r.

$$\boxed{\begin{aligned} \hat{i}_r &= \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \\ \hat{i}_\theta &= -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} \\ \vec{r} &= r\,\hat{i}_r \end{aligned}}$$

Example: Place a charge q_1 at the origin. Then the force $\vec{F}_2(\vec{r})$ on a second charge q_2 located at $\vec{r}_2$ is given in polar coordinates by

Note absence of absolute value

$$\vec{F}_2(\vec{r}_2) = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_2^2}}_{\text{depends only on } r}\hat{\imath}_r \equiv F_2(r_2)\,\hat{\imath}_r$$



$$\vec{F}_2(\vec{r}_2) = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_2^2}\hat{\imath}_r$$

$$\vec{F}_3(\vec{r}_3) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_3^2}\hat{\imath}_r$$

But wait... didn't I tell you previously to ignore the signs when computing the magnitudes of electric forces?

Yes! Right now we are considering the complete vector notation for a very specific purpose...

(*) Any force of the form $\vec{F}(\vec{r}) = F(r)\,\hat{\imath}_r$ is <u>conservative</u> and has a corresponding <u>potential energy</u> function.

1.3.b) Potential Energy Functions

* Potential energy was a key concept from Mechanics

* Potential energy functions make our lives much simpler because

  (a) Potential energy is a scalar, not a vector (like forces)

  (b) We can typically exploit energy conservation:

$$\underbrace{KE(\vec{r}_1)}_{\text{Kinetic Energy}} + \underbrace{U(\vec{r}_1)}_{\text{Potential Energy}} = KE(\vec{r}_2) + U(\vec{r}_2)$$

How do we compute potential energy functions from a force?

$$\boxed{U(b) - U(a) = -\int_a^b \vec{F} \cdot d\vec{r}}$$

($\vec{F}$ must be a conservative force)

* Strictly speaking, only changes in potential energy are physically meaningful.

* But it is convenient to choose a specific point p where $U(p) = 0$ (example: choose ground level to be the location where gravitational potential energy is 0)

Recall that $-\int_a^b \vec{F}\cdot d\vec{r}$ is called a "line integral".

To evaluate the line integral (i.e., potential energy difference), we need:

(1) The force $\vec{F}(\vec{r})$

(2) A path that starts at position "a" and ends at position "b"

To obtain the potential energy function, we need one more thing:

(3) The location p at which we define $U(p) \equiv 0$.

⊛ To calculate the difference in potential energy between two points, we don't need to specify p.

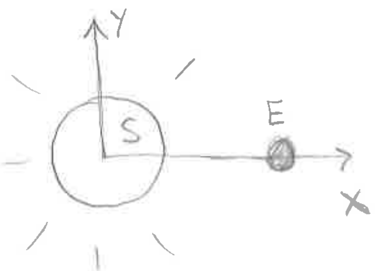
⊛ For simple paths along the x, y, r, or θ directions, it is very useful to use the following:

From $\vec{v} = \frac{d\vec{r}}{dt}$ we see that $d\vec{r} = \vec{v}\,dt$

Cartesian: $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \Rightarrow \boxed{d\vec{r} = dx\,\hat{i} + dy\,\hat{j}}$

Polar: $\vec{v} = \frac{dr}{dt}\hat{i}_r + r\frac{d\theta}{dt}\hat{i}_\theta \Rightarrow \boxed{d\vec{r} = dr\,\hat{i}_r + r\,d\theta\,\hat{i}_\theta}$

Let's put all of this together to calculate the gravitational potential energy <u>function</u> for the Earth rotating around the Sun:



$$\vec{F}_E(\vec{r}) = -G\frac{m_S m_E}{r^2}\,\hat{\imath}_r$$

Question: Where to define $U(p) \equiv 0$?

Answer: Most convenient if we choose $U(r=\infty) \equiv 0$,

because the Earth and Sun feel no force very far apart.

Then $U(r) = U(r) - 0 = U(r) - U(\infty)$ } Convenient to choose straight line path:

$$= -\int_{\infty}^{r} \vec{F}_E(\vec{r})\cdot d\vec{r} = -\int_{\infty}^{r}\left(-G\frac{m_S m_E}{r^2}\hat{\imath}_r\right)\cdot\left(dr\,\hat{\imath}_r + r\,d\theta\,\hat{\imath}_\theta\right)$$

$$= \int_{\infty}^{r} G\frac{m_S m_E}{r^2}\,dr \qquad \begin{pmatrix} \hat{\imath}_r\cdot\hat{\imath}_r = 1 \\ \hat{\imath}_r\cdot\hat{\imath}_\theta = 0 \end{pmatrix}$$

$$= G m_S m_E \left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$\boxed{U(r) = -G\frac{m_S m_E}{r}}$$ Note that this is a scalar.

Key Properties of Potential Energy Functions

P1) The force can always be obtained from the potential energy by taking partial derivatives

$$U(x,y) \longrightarrow F_x = -\frac{\partial U}{\partial x} \quad \text{and} \quad F_y = -\frac{\partial U}{\partial y}$$

Example: $U(x,y) = x^2 + xy^2$

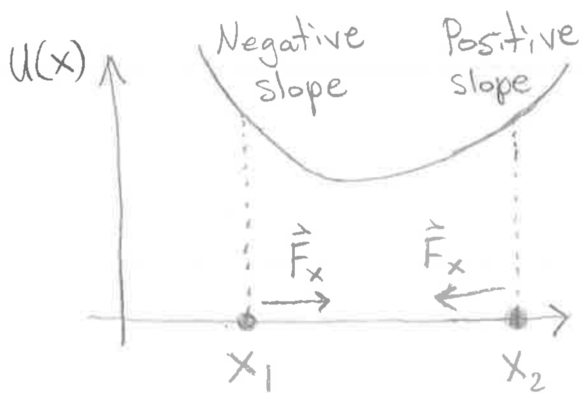
Then $F_x = -\dfrac{\partial U}{\partial x} = -2x - y^2$ (y treated as constant)

$F_y = -\dfrac{\partial U}{\partial y} = -2xy$ (x treated as constant)

$$\vec{F}(x,y) = F_x\,\hat{\imath} + F_y\,\hat{\jmath}$$

P2) Objects always try to lower their potential energy (water flows downhill, etc.).

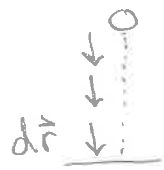
Why? Because that is the direction of the associated force.



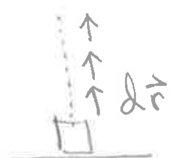
At x_1: $F_x = -\dfrac{\partial U}{\partial x} = -(-\#) > 0$

At x_2: $F_x = -\dfrac{\partial U}{\partial x} = -(+\#) < 0$

P3) If an object moves in the direction of the force, the object's potential energy decreases

Example: Drop a ball.  $\downarrow \vec{F}_g$ $\left(-\vec{F}_g \cdot d\vec{r} < 0\right)$

Likewise, if an object is moved against a force, then its potential energy increases

Example: Lift a box.  $\downarrow \vec{F}_g$ $\left(-\vec{F}_g \cdot d\vec{r} > 0\right)$

(*) Important Point: It does not matter that you are providing an upward force. Gravity is the conservative force, and it points opposite the motion.

For completeness, what is an example of a non-conservative force?

$\longrightarrow$ Consider $\vec{F}(\vec{r}) = \alpha r \hat{\imath}_\theta$, where α is a positive constant.

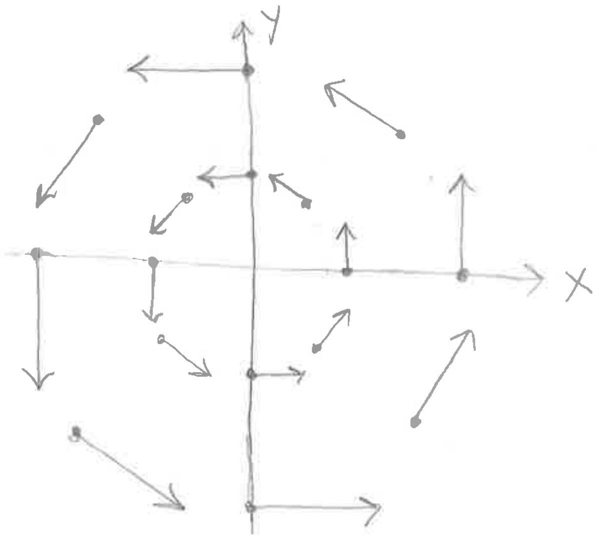
How should we visualize this force?

$\vec{F}(\vec{r}) = \alpha r \hat{\iota}_r$

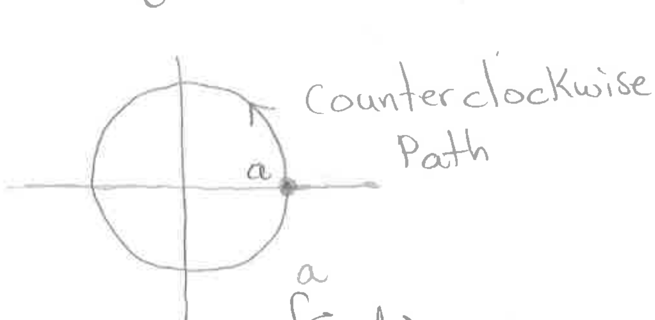
This tells us that the magnitude of the force increases with the distance from the origin.

$\vec{F}(\vec{r}) = \alpha r \hat{\iota}_\theta$

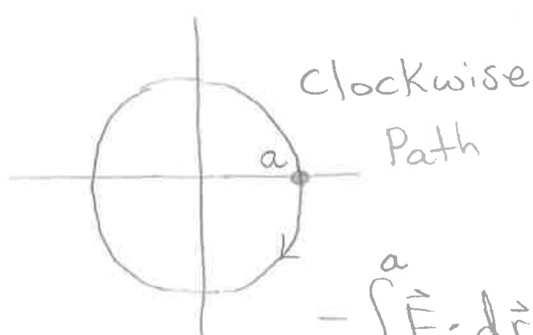
This tells us that the force is always in the tangential direction.



Note that $-\int_a^b \vec{F}\cdot d\vec{r}$ is not uniquely defined (the line integral will depend on path) and therefore $U(r)$ is ill-defined.



$$-\int_a^a \vec{F}\cdot d\vec{r} < 0$$



$$-\int_a^a \vec{F}\cdot d\vec{r} > 0$$