8 Major Topics

① Mathematics (calculus) of motion

$$\begin{tikzcd} a(t) \arrow[r, bend left, "\text{integrate}"] & v(t) \arrow[l, bend left, "\text{differentiate}"] \arrow[r, bend left, "\text{integrate}"] & x(t) \arrow[l, bend left, "\text{differentiate}"] \end{tikzcd}$$

$$x(t) = \int v(t)\,dt \leftrightarrow v = \frac{dx}{dt}$$

$$v(t) = \int a(t)\,dt \leftrightarrow a = \frac{dv}{dt}$$

② Newton's 2nd and 3rd Laws (now including massive pulleys $\longrightarrow$ use $\tau = I\alpha$)

* For each object in motion, use either

$\vec{F}_{tot} = m\vec{a}$ or $\tau = I\alpha$ $(a = \alpha R)$

usually just $\tau = FR$ for pulleys

* Use for time-dependent forces $F(t)$.

③ Work-Kinetic Energy Theorem:

$$\frac{1}{2}mv^2(\vec{r}_2) - \frac{1}{2}mv^2(\vec{r}_1) = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{tot} \cdot d\vec{r}$$

* Use especially for nonconservative friction forces. And <u>always</u> for x-dependent friction forces $\mu(x)$.

* W-KE gives $v(x)$. <u>No</u> <u>time</u> <u>dependence</u>!

(4) Conservation of energy: for position-dependent forces $F(x)$, obtain $v(x)$ from

(a) Find $U(x) = -\int F(x)\,dx$ $\quad \vec{F} = F(x)\,\hat{i}_x$.

(b) $KE(x_2) + U(x_2) = KE(x_1) + U(x_1)$

(5) Conservation of momentum: for collisions or explosions.

$$\vec{P}^{\,i}_{total} = \vec{P}^{\,f}_{total} \quad \begin{cases} p_x^i = p_x^f \\ p_y^i = p_y^f \end{cases}$$

(6) "Inverse Newton's Laws": from $\{r(t), \omega(t)\}$ find $\vec{a} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{i}_r + \left(\alpha r + 2\omega\frac{dr}{dt}\right)\hat{i}_\theta$.

From $\vec{a}$ find $\vec{F}_{tot} = m\vec{a}$ and equate to underlying physical forces.

(7) Conservation of angular momentum: for collisions involving rotation or changing moment of inertia.

$\vec{L}_i = \vec{L}_f$ where $\vec{L} = \vec{r} \times \vec{p}$ or $L = I\omega$.

⑧ Simple Harmonic Motion: for time dependence of spring motion.

① $\vec{F} = m\vec{a} = m\dfrac{d^2\vec{x}}{dt^2}$ 　　springs $(F = -Kx)$

② $\dfrac{d^2x}{dt^2} = -\left(\dfrac{K}{m}\right)x$

$\dfrac{K}{m} = \omega^2$

③ Most general solution is

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

Find A and B from initial conditions $x(0)$ and $v(0)$.

(*) One full oscillation occurs when $\omega t = 2\pi$

$$\Rightarrow \quad t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

Likewise for half or quarter motion.