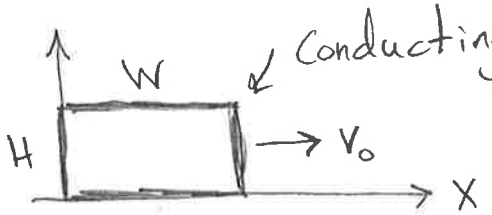
Example (Exam 3, 2012, Q2)



Background magnetic field is

$\vec{B} = B_0(\alpha+\beta x)$ into the page.

At $t=0$ the magnetic flux is

$$\phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA = \int_0^W B_0(\alpha+\beta x)\,dx \int_0^H dy$$

$$= B_0\left(\alpha W + \tfrac{1}{2}\beta W^2\right) H$$

Resistance of loop: $R = \frac{\rho L}{A} = \frac{\rho(2W+2H)}{a}$ ← perimeter

Induced emf when loop moves with speed V_0:

$$\phi_B(t) = \int_{0+V_0t}^{W+V_0t} B_0(\alpha+\beta x)\,dx \int_0^H dy$$

↖ limits change with time

$$= HB_0\left(\alpha(W+V_0t) - \alpha V_0 t + \tfrac{1}{2}\beta(W+V_0t)^2 - \tfrac{1}{2}\beta V_0^2 t^2\right)$$

$$= HB_0\left[\alpha + \tfrac{1}{2}\beta W^2 + \beta W V_0 t\right]$$

$$\Rightarrow \mathcal{E} = -\frac{d\phi_B}{dt} = -HB_0\beta W V_0$$

$$\mathcal{E} = iR$$

$$HB_0\beta W v_0 = i\left(\frac{\rho(2W+2H)}{a}\right)$$

$$i = \frac{HWB_0\beta v_0 a}{2\rho(W+H)}$$

Direction of current from 5-steps:

$\phi_B^i = \otimes$

$\phi_B^f = \otimes$ since B increases in +x direction

$\Delta\phi_B = \otimes - \text{⊙} = \otimes$

$-\Delta\phi_B = \odot$

$\vec{B}_{ind}$ is in $\odot$ direction if i is counterclockwise