Example (Exam 3, 2012, Q4)

A cylindrical wire of radius $a$, length $L$, and resistivity $\rho_1$ is surrounded by a second wire with inner radius $a$, outer radius $b$, length $L$, and resistivity $\rho_2$. They are attached lengthwise to a battery with voltage $V$.

(a) If the inner cylinder carries a current $i_1$ uniformly spread over its cross-sectional area, find $\vec{B}$ inside.

(a) Solution: From Ampere's Law, we draw a circular Amperian Loop inside the wire and concentric

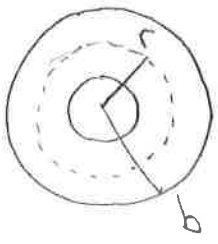

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$$

$$B(2\pi r) = \mu_0 (J_1 A_{enc})$$

$$B(2\pi r) = \mu_0 \left(\frac{i}{\pi a^2}\right)(\pi r^2)$$

$$\boxed{B = \frac{\mu_0 i}{2\pi a^2} r}$$

(b) If the outer wire has current $i_2$ spread uniformly over its cross-sectional area, find $\vec{B}$ inside this wire?




$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$$

both currents now enclosed by loop.

$$B(2\pi r) = \mu_0 (i_1 + J_2 A_{enc})$$

Now $J_2 = \dfrac{i_2}{\pi(b^2-a^2)}$ and $A_{enc} = \pi(r^2-a^2)$

$$\Rightarrow \quad B(2\pi r) = \mu_0\left(i_1 + \frac{i_2}{\pi(b^2-a^2)}\pi(r^2-a^2)\right)$$

$$\boxed{B = \frac{\mu_0}{2\pi r}\left[i_1 + i_2\frac{r^2-a^2}{b^2-a^2}\right]}$$