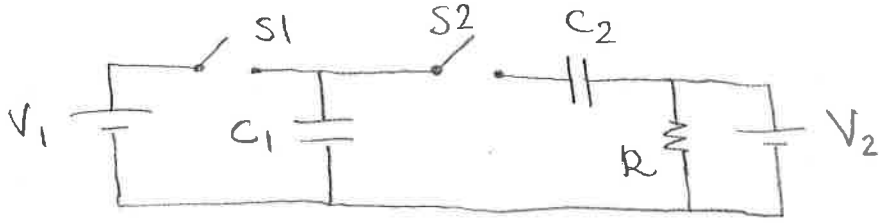
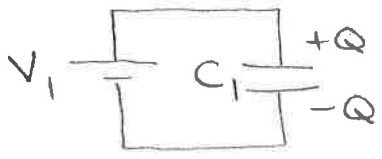
Example (Exam 2, 2012, Q3)

In the circuit shown below, switch 1 has been closed for a long time and switch 2 has been open a long time. The two switches are then flipped simultaneously. Find all resulting currents through resistors and charges on capacitors after the system has reached a steady state.



Solution:

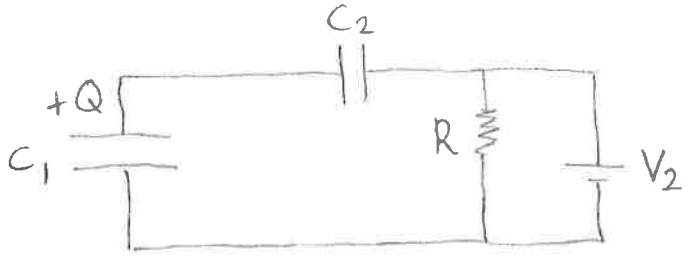
In the initial configuration charge builds up on capacitor C_1 but not on capacitor C_2, which is not in a closed circuit. We find the initial charge Q from Kirchhoff's loop rule



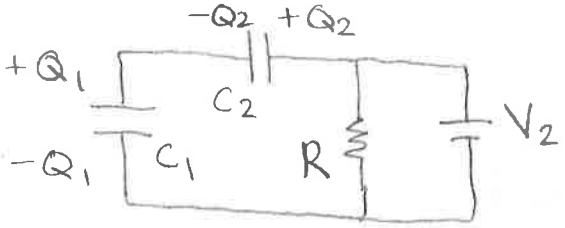
$$V_1 - \frac{Q}{C_1} = 0$$

$$\Rightarrow \underline{Q = V_1 C_1}$$

What happens to this charge after the switches are flipped? Note that the +Q charge is now trapped between the two capacitors:



this $+Q$ charge can be redistributed between C_1 and C_2, but it cannot leave the system



Note: $\underline{Q_1 - Q_2 = Q}$

Use Kirchhoff's loop rule twice:

(Outer loop counterclockwise) $\rightarrow$ $V_2 - \frac{Q_2}{C_2} - \frac{Q_1}{C_1} = 0$

$$\Rightarrow V_2 = \frac{Q_2}{C_2} + \frac{Q+Q_2}{C_1} = \frac{Q_2}{C_2} + \frac{V_1 C_1 + Q_2}{C_1}$$

$$\Rightarrow V_2 - V_1 = Q_2\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Rightarrow \boxed{Q_2 = \frac{C_1 C_2}{C_1 + C_2}\left(V_2 - V_1\right)}$$

$$\Rightarrow Q_1 = Q + Q_2 = C_1 V_1 + \frac{C_1 C_2}{C_1 + C_2}\left(V_2 - V_1\right)$$

$$\Rightarrow \boxed{Q_1 = \frac{C_1}{C_1 + C_2}\left(C_1 V_1 + C_2 V_2\right)}$$