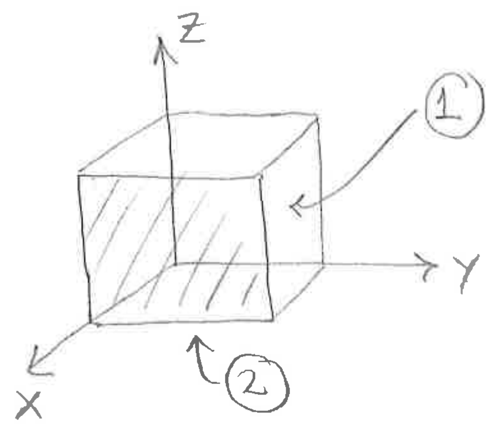
Example: (Exam 1, 2012, Q3)

A cube with edge length "a" has one corner at the origin as shown in the figure. The electric field in this region of space is given by $\vec{E} = \alpha x^2 \hat{i}_x$. Find the flux through faces 1 and 2.



Solution:

Flux through surface (1)

(1) $\hat{n} = \hat{i}_y$

(2) $\vec{E} \cdot \hat{n} = (\alpha x^2 \hat{i}_x) \cdot (\hat{i}_y)$

$= 0$

$$\Rightarrow \int_{1} \vec{E} \cdot d\vec{A} = \int_{1} (\vec{E} \cdot \hat{n}) \, dA = \int 0 \, dA = 0$$

* Same holds for three other sides, but not for the front face (2) and the opposite back face.

Flux through face (2)

(1) $\hat{n} = \hat{i}_x$

(2) $\vec{E} \cdot \hat{n} = (\alpha x^2 \hat{i}_x) \cdot (\hat{i}_x) = \alpha x^2$

(3) on the surface $x = a$

$$\Rightarrow \vec{E} \cdot \hat{n} = \alpha x^2 = \alpha a^2$$

(4) $dA = dy\,dz$ and $\vec{E} \cdot \hat{n}$ is independent of both y and z

(5) $$\int_{2} \vec{E} \cdot d\vec{A} = \int \alpha a^2 \, dA = \alpha a^2 \int_0^a dy \int_0^a dz$$

$$= \alpha a^2 \left(a^2\right) = \boxed{\alpha a^4}$$

Finally for the back surface opposite face 2 we note that $x = 0$ on this surface. Therefore

$$\vec{E} \cdot \hat{n} = \alpha x^2 \rightarrow \alpha (0)^2 = 0$$

Total electric flux through the closed cube is therefore

$$\oint \vec{E} \cdot d\vec{A} = \underbrace{0 + 0 + 0 + 0 + 0}_{5 \text{ faces}} + \alpha a^4$$

From Gauss's Law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\varepsilon_0}$

$$\Rightarrow \boxed{q_{enc} = \alpha a^4 \varepsilon_0}$$