Example: (Exam 1, 2013 Q3)

A solid sphere of copper has been given a charge Q. The sphere has radius R. The electric field is found to be zero inside ($r<R$) and given by $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ outside ($r>R$) with direction radially outward. Find $V(3R)-V(0)$.

Solution: Given the electric field, we compute the potential difference $V(3R)-V(0) = -\int_0^{3R} \vec{E}\cdot d\vec{r}$.

Since $\vec{E}$ is radially outward for $r>R$ and zero for $r<R$,

$$-\int_0^{3R} \vec{E}\cdot d\vec{r} = -\int_0^{R} \vec{0}\cdot d\vec{r} - \int_R^{3R} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{i}_r \cdot \left(dr\,\hat{i}_r + r\,d\theta\,\hat{i}_\theta\right)$$

($d\theta \to 0$)

$$= -\int_R^{3R} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= \frac{-Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^{3R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{3R}-\frac{1}{R}\right]$$

$$= \frac{-2Q}{12\pi\varepsilon_0 R} = \boxed{\frac{-Q}{6\pi\varepsilon_0 R}}$$

(*) Note that $r=0$ is at a higher electric potential, even though $\vec{E}=0$ at $r=0$.