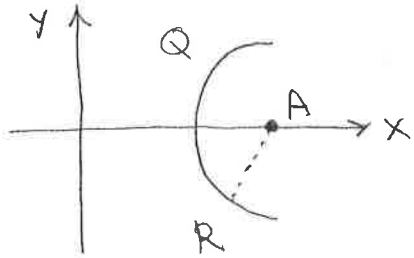
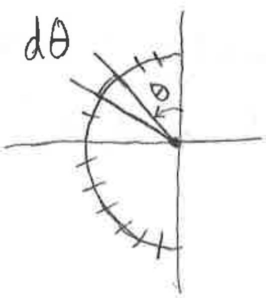
Example (Exam 1, 2015, Q2)

An amount of charge Q is uniformly spread over a semi-circle of radius R whose center is located a distance A from the origin. What point charge would have to be placed at the origin so that the $\vec{E}$ field at the center of the circle is 0?



Solution: First, calculate the $\vec{E}$ field due to the charge distribution. No worries, just follow exactly the 5-step process.

(1) Break up charge into small pieces



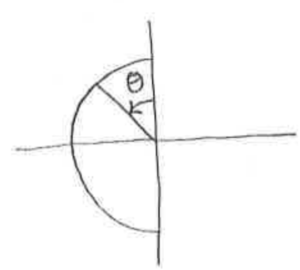
$$dq = \lambda\, ds \quad \text{(here, let's choose } ds = R\,d\theta)$$

$$dq = \left(\frac{Q}{\pi R}\right)(R\,d\theta)$$

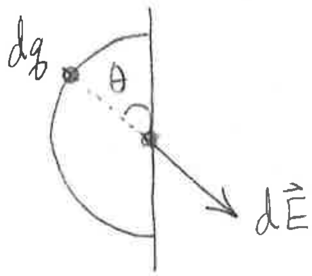
$\frac{\text{total charge}}{\text{total length}}$ for uniform charge

$$\boxed{dq = \frac{Q}{\pi}\,d\theta}$$

(2) Define integration region : $\int_0^{\pi} d\theta$



③ Arbitrary point θ within $0 \le \theta \le \pi$:

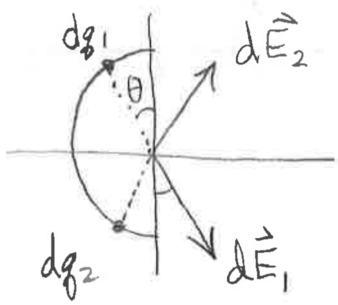


Magnitude: $dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2}$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{(Q/\pi)\,d\theta}{R^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\pi R^2}\,d\theta$$

④ Break into components and look for symmetries :



Symmetric points give y-components that <u>cancel</u>.

$\Rightarrow$ Only compute dE_x.

$$dE_x = \left(\frac{1}{4\pi\varepsilon_0} \frac{Q}{\pi R^2}\,d\theta\right) \sin\theta$$

see geometry in figure to get $\sin\theta$, not $\cos\theta$.

⑤ Integrate components separately : (E_y vanishes from symmetry)

$$E_x = \int dE_x = \int_0^{\pi} \frac{1}{4\pi\varepsilon_0} \frac{Q}{\pi R^2} \sin\theta\,d\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{\pi R^2} \int_0^{\pi} \sin\theta\,d\theta = \frac{Q}{4\pi^2 R^2 \varepsilon_0} \left(-\cos\theta\right)_0^{\pi} = \frac{Q}{2\pi^2 R^2 \varepsilon_0}$$

What charge at the origin will balance this $\vec{E}$ field?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{A^2}$$ (in $-\hat{\imath}_x$ direction

$\Rightarrow$ q must be negative)

$$\Rightarrow \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{q}{A^2}}_{\text{point charge}} + \underbrace{\frac{Q}{2\pi^2 R^2 \varepsilon_0}}_{\text{semicircle charge}} = 0$$

$$\Rightarrow \boxed{q = \frac{-2QA^2}{\pi R^2}}$$