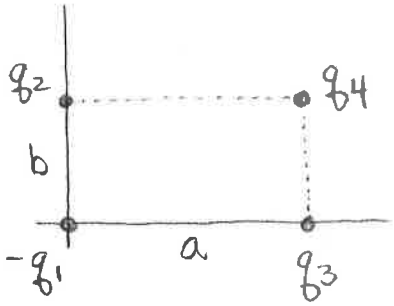
Example (Exam 1, 2015, Q1)

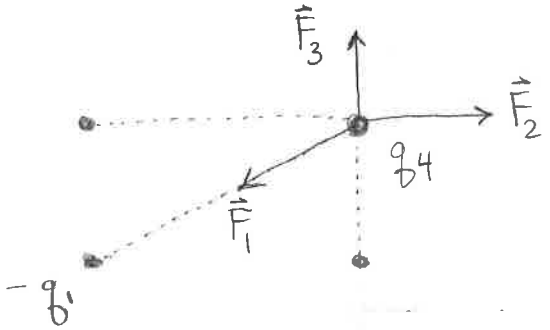


The distances a and b are known. The charge $-q_1$ is known, but both q_2 and q_3 are unknown. What are these charges if the total force on the positive q_4 charge is 0?

Solution:

(1) Identify the "type" of problem. This is clearly a Coulomb Law electric force question: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$.

(2) We are looking for the total force on q_4. There are three forces acting on q_4. Start by drawing them:



Since $\vec{F}_1$ is attractive, $F_{1x} < 0$ and $F_{1y} < 0$.

Therefore, both q_2 and q_3 must be positive charges so that $F_{2x} > 0$ and $F_{3y} > 0$, giving $\vec{F}_{total} = 0$.

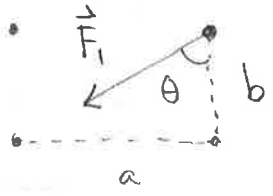
(3) Compute magnitudes of all forces:

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_4}{a^2+b^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_4}{a^2}$$

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_4}{b^2}$$

(4) Resolve all vectors into their x and y components:



$$F_{1x} = (-)\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_4}{a^2+b^2}\sin\theta$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_4}{a^2+b^2}\cdot\frac{a}{\sqrt{a^2+b^2}}$$

$$F_{1y} = (-)\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_4}{a^2+b^2}\cos\theta$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_4}{a^2+b^2}\cdot\frac{b}{\sqrt{a^2+b^2}}$$

$$F_{2x} = (+)\frac{1}{4\pi\varepsilon_0}\frac{q_2 q_4}{a^2}$$

$$F_{3y} = (+)\frac{1}{4\pi\varepsilon_0}\frac{q_3 q_4}{b^2}$$

(5) Add up x and y components separately:

$$\vec{F}_{total} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2 q_4}{a^2} - \frac{q_1 q_4}{(a^2+b^2)^{3/2}}\,a\right)\hat{\imath} + \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3 q_4}{b^2} - \frac{q_1 q_4}{(a^2+b^2)^{3/2}}\,b\right)\hat{\jmath}$$

For total force to be 0, solve

$$\frac{q_2 q_4}{a^2} - \frac{q_1 q_4}{(a^2+b^2)^{3/2}}\,a = 0$$

$$\Rightarrow \boxed{q_2 = \frac{a^3}{(a^2+b^2)^{3/2}}\,q_1}$$

$$\frac{q_3 q_4}{b^2} - \frac{q_1 q_4}{(a^2+b^2)^{3/2}}\,b = 0$$

$$\Rightarrow \boxed{q_3 = \frac{b^3}{(a^2+b^2)^{3/2}}\,q_1}$$

Focus Question: Exam 1, 2012, Q1