Exam 3 Review

Five Major Topics:

(1) Magnetic Forces (on moving charges, on current-carrying wires)

(2) Biot-Savart Law (how currents produce $\vec{B}$ fields)

(3) Ampere's Law (finding $\vec{B}$ field near symmetrical currents)

(4) Faraday's Law (finding induced EMF and current due to changing magnetic flux

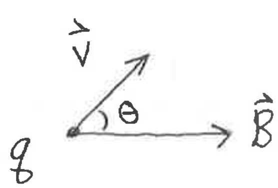
(5) Time-dependent circuits (inductors in circuits, derive differential equations from Kirchhoff's loop rule)

① Magnetic Forces

(a) Point particles: $\vec{F}_B = q\vec{v} \times \vec{B} = qvB\sin\theta$ (RHR)

Here θ is the angle between $\vec{v}$ and $\vec{B}$ when they are drawn tail-to-tail

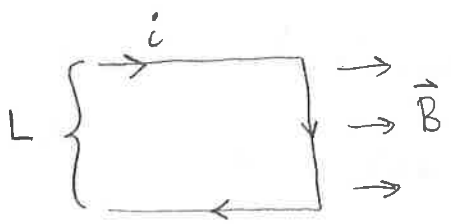
Right-hand-rule: cross product gives a vector that is perpendicular to both $\vec{v}$ and $\vec{B}$ (fingers in direction of first vector $\vec{v}$, curl toward $\vec{B}$, and thumb is $\vec{v}\times\vec{B}$ direction)



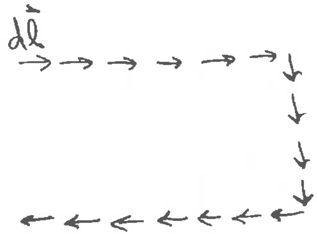
$\vec{F} = qvB\sin\theta$ (into page)

(b) Magnetic Forces on currents :

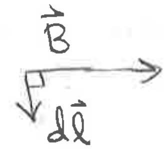
$d\vec{F} = i\, d\vec{l} \times \vec{B}$ Need to integrate to get total force



$$\vec{F} = \int_0^L i\, d\vec{l} \times \vec{B}$$ (For horizontal sections of wire, $d\vec{l} \times \vec{B} = 0$.)



$$= i \int_0^L dl\, B \sin\theta$$



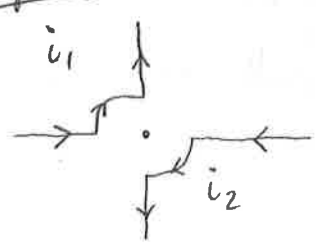
$$= i B \int dl = iBL$$ (out of page from RHR)

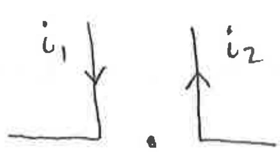
(2) Biot-Savart Law

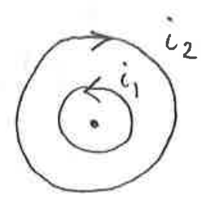
(*) Whenever you see current wires that are curved or broken into segments, find $\vec{B}$ field at a location using

$$\int d\vec{B} = \int \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} = \int \frac{\mu_0 i}{4\pi} \frac{dl \sin\theta}{r^2}$$ (RHR for direction)

Examples







(*) Either r or $\sin\theta$ may depend on integration variable l.

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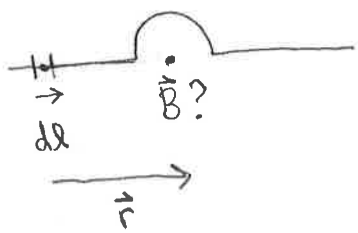
Wires directed toward or away from location of $\vec{B}$ field

do not contribute since $d\vec{l} \times \underbrace{\vec{r}}_{} = 0$

vector from $d\vec{l}$ segment to location of $\vec{B}$ field.



For circular current segments $\frac{d\vec{l} \times \hat{r}}{r^2} = \frac{dl \sin 90^\circ}{R^2} = \frac{dl}{R^2}$

when $\vec{B}$ is evaluated at a point on axis through center

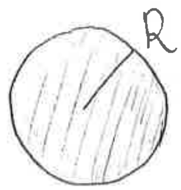
(*) For multiple wires, always compute $\vec{B}$ separately for each and add up as vectors.

(3) Ampere's Law

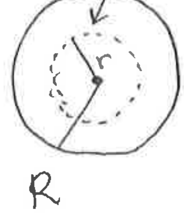
Always use when you see a thick wire or very long straight wires.

Always draw out your Amperian loop (usually it will be a circle) centered on the current's axis of symmetry.

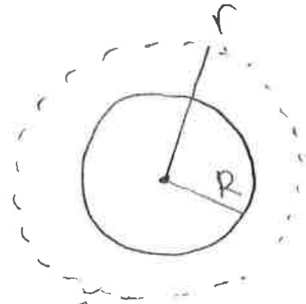
Examples



total current i through thick wire into page



Amperian loop $(r<R)$ to find $\vec{B}$ inside thick wire



Amperian loop $(r>R)$ to find $\vec{B}$ outside

$(r<R)$: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$ ← total current that passes through surface inside the loop

$$B(2\pi r) = \mu_0 \left(\underset{\substack{\uparrow \\ \text{current density}}}{J} \cdot \underset{\substack{\uparrow \\ \text{current area enclosed}}}{A_{enc}}\right)$$

$$B(2\pi r) = \mu_0 \underbrace{\left(\frac{i}{\pi R^2}\right)}_{J}\underbrace{\left(\pi r^2\right)}_{\text{loop area}} \quad \Rightarrow \quad B = \frac{\mu_0 i}{2\pi}\frac{r}{R^2}$$

$(r>R)$

$$B(2\pi r) = \mu_0 \underbrace{\left(\frac{i}{\pi R^2}\right)}_{J}\underbrace{\left(\pi R^2\right)}_{\text{current area enclosed}} \quad (\text{not } \pi r^2 \text{!!})$$

$$B = \frac{\mu_0 i}{2\pi r} \quad (\text{clockwise direction})$$

⊛ $\vec{B}$ fields <u>circulate</u> around according to RHR: thumb in i direction and fingers curl around in $\vec{B}$ direction

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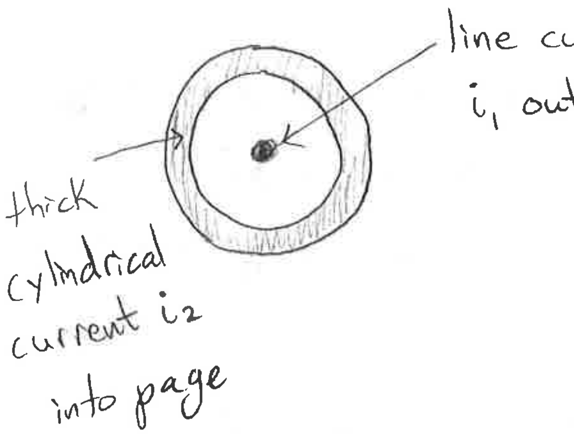
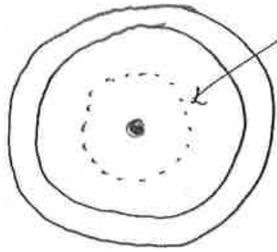
For I_{enc}, currents going in and currents going out get <u>opposite signs</u>:



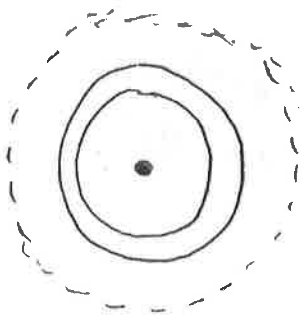


$\vec{B}$ field inside:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 i_1$$

only i_1 goes through Amperian surface



$\vec{B}$ field outside:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 (i_1 - i_2)$$

(*) <u>Nonuniform</u> current densities: $I_{enc} = \int J(r)\, dA$ ... circular cross section wire

$$= \int J(r)\,[2\pi r\, dr]$$

(4) Faraday's Law

A changing magnetic flux through surface of a closed wire loop produces an EMF:

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

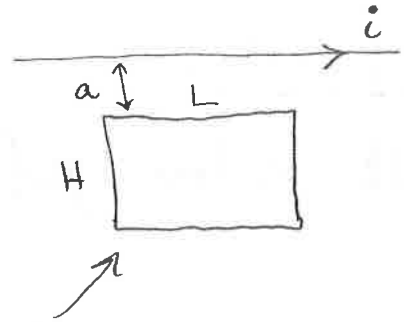
* The minus sign only tells us direction of induced current (4-step process)

ϕ_B^i	ϕ_B^f	$\Delta\phi_B$	$-\Delta\phi_B$
1	2	3	4

Induced current must be such that it produces a $\vec{B}$ field in $-\Delta\phi_B$ direction.

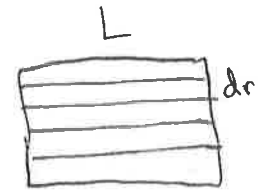
* If $\vec{B}$ changes in space, $\oint \vec{B}\cdot d\vec{A}$ will be nontrivial

$\boxed{\text{Examples}}$



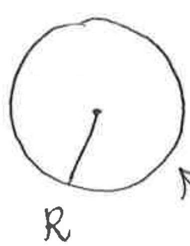
Current produces

$$B(r) = \frac{\mu_0 i}{2\pi r}$$



Break up wire loop into segments $dA = L\,dr$

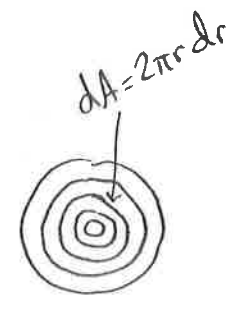
$$\Rightarrow \int \vec{B}\cdot d\vec{A} = \int_a^{a+H} \frac{\mu_0 i}{2\pi r}\,(L\,dr)$$



$\vec{B}(r)$ varies radially

Break up wire loop into segments

$$dA = 2\pi r\,dr$$



$$\int \vec{B}\cdot d\hat{A} = \int_0^R B(r)\,[2\pi r\,dr]$$