EXAM II Physics 206 FALL 2020

Last Name.. First.................................... Section Number..........

USEFUL EQUATIONS

If $f(x) = a\,x^n$, then

$$\frac{df}{dx} = n\,a\,x^{n-1}$$

$$\int f(x)\,dx = \frac{a}{n+1}\,x^{n+1} + C$$

Work – Kinetic Energy Theorem:

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{tot} \cdot d\vec{s} = \frac{1}{2}mv^2(\vec{r}_2) - \frac{1}{2}mv^2(\vec{r}_1).$$

If $\vec{F}$ is conservative, then there exists a potential energy function U such that

$$U(\vec{r}_2) - U(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

and

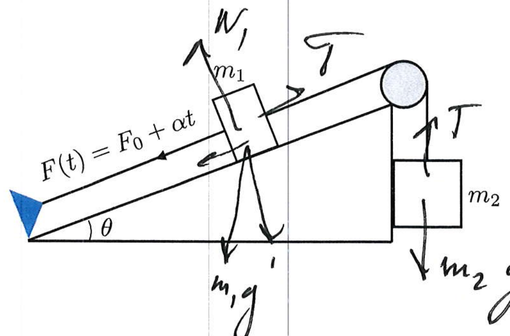
$$F_x = -\frac{\partial U}{\partial x}, \quad F_y = -\frac{\partial U}{\partial y}.$$

Note: The symbol g stands for the **magnitude** of the acceleration due to gravity, and therefore it is always a positive quantity.

Free-body force diagrams are very important!

Do not spend too much time on algebra!

1. (25 points) Two blocks of mass m_1 and m_2 are held at rest by a wire directed down a frictionless inclined plane as shown in the figure. At times $t < 0$ the wire provides a constant force F_0. (i) What value of F_0 is needed so that both blocks remain at rest? At time $t = 0$ the external force starts to increase according to $F(t) = F_0 + \alpha t$, where α is a positive constant. (ii) What is the speed of block m_2 as a function of time?



Law

4

$F = ma$

Application

(I)

$$F_0 + m_1 g \sin\theta - T = 0$$
$$+ \quad -m_2 g + T = 0$$

$$F_0 + m_1 g \sin\theta - m_2 g = 0 \qquad F_0 = g(m_2 - m_1 \sin\theta)$$

(II) Way 1

$$(A) \int_0^x (F_0 + \alpha t)\,dx = \frac{(m_1+m_2)}{2} v^2 + m_2 g x - m_1 g x \sin\theta$$

$$dx = v\,dt$$

$$\frac{d(A)}{dt} \Rightarrow (F_0 + \alpha t) v = (m_1 + m_2) v \cdot \frac{dv}{dt} + (m_2 g - m_1 g \sin\theta) v$$

$$\frac{dv}{dt} = \frac{F_0 + \alpha t - g(m_2 - m_1 \sin\theta)}{m_1 + m_2}$$

$$V(0) = 0 \qquad \boxed{V = \int \frac{dv}{dt} dt = \frac{F_0 - g(m_2 - m_1 \sin\theta)}{m_1 + m_2} t + \frac{\alpha}{2(m_1+m_2)} t^2 + 0}$$

Way 2

$$(F_0 + \alpha t) + m_1 g \sin\theta - T = m_1 a$$
$$+ \quad -m_2 g + T = m_2 a$$

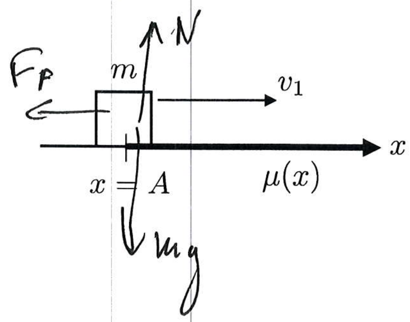
$$F_0 + \alpha t + m_1 g \sin\theta - m_2 g = (m_1 + m_2) a$$

$$a = \frac{dV}{dt} \qquad V = \int a\,dt = \frac{F_0 + (m_1 \sin\theta - m_2) g}{m_1 + m_2} t + \frac{\alpha}{(m_1+m_2)} \frac{t^2}{2} + \ldots$$

$$V(0) = 0$$

Result

2. (25 points) An object of mass m starts at $x = A$ with a speed v_1 in the $+x$ direction. For $x > A$ the surface has a coefficient of friction $\mu(x) = \frac{\alpha}{x^2}$, where α is a positive constant. Calculate the work done by friction from $x = A$ to an arbitrary point $x = B$ along the box's path of motion. What is the object's kinetic energy at $x = B$?



Law

$$W = \int_{V_1}^{V_2} \vec{F}\, d\vec{s}$$

Application

$$N = mg \qquad F_F = \mu N = \mu mg$$

$$W_F = \int_A^B -F_F\, dx = \int_A^B -\frac{\alpha}{x^2}\, mg\, dx =$$

Sign

$$= \frac{\alpha mg}{x}\Big|_A^B = \alpha mg\left(\frac{1}{B} - \frac{1}{A}\right)$$

$$W = \Delta K \qquad K_I = \frac{mV_1^2}{2} \qquad K_F = \frac{mV_B^2}{2}$$

$$\Delta K = K_F - K_I = \frac{mV_B^2}{2} - \frac{mV_1^2}{2} = \alpha mg\left(\frac{1}{B} - \frac{1}{A}\right)$$

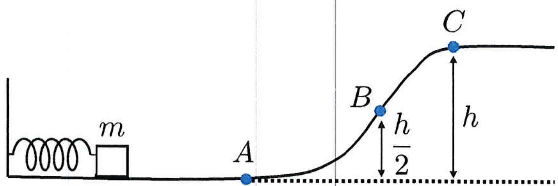
$$\boxed{\frac{mV_B^2}{2} = \frac{mV_1^2}{2} + \alpha mg\left(\frac{1}{B} - \frac{1}{A}\right)}$$

Result

3. (25 points) A small box with mass m is released from rest by a spring compressed a distance d. The spring constant is k. After being released, the object slides along a frictionless track. What is the speed of the box when it reaches point B at height $h/2$? What must be the value of d so that the box comes to rest at the top of the track at point C?

Law

$\Delta E = 0$ 5



Application

$$\frac{kd^2}{2} = \frac{mV^2}{2} + mgy$$

3 3 3

$y = h/2$ 3

$$V = \sqrt{\frac{kd^2}{m} - 2g\frac{h}{2}} = \boxed{\sqrt{\frac{kd^2}{m} - gh}}$$ 1

$V = 0 \quad y = h$ 3

$$\frac{kd^2}{2} = mgh \qquad \boxed{d = \sqrt{\frac{2mgh}{k}}}$$ 1

or 3

Result

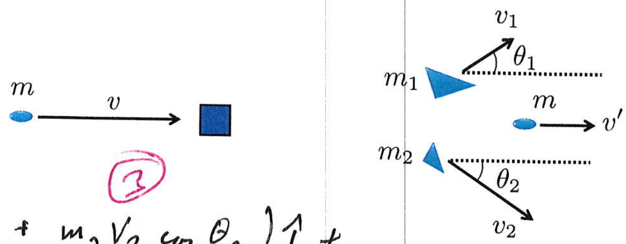
4. (25 points) A bullet of mass m travels with speed v towards a block that is initially at rest. The bullet cuts the block into two pieces with masses m_1 and m_2. After the impact, the bullet continues on its original path but with a reduced speed v', while masses m_1 and m_2 travel with speeds v_1 and v_2 at the angles θ_1 and θ_2. If θ_1 and θ_2 are known, find the values of v_1 and v_2.

Law

3

$$\vec{P_i} = \vec{P_f}$$

Application



$$mV\,\hat{i} = \left(m_1 V' + m_1 V_1 \cos\theta_1 + m_2 V_2 \cos\theta_2\right)\hat{i} + \left(m_1 V_1 \sin\theta_1 - m_2 V_2 \sin\theta_2\right)\hat{j}$$

$$m_1 V_1 \sin\theta_1 = m_2 V_2 \sin\theta_2 \qquad m_1 V_1 = m_2 V_2 \frac{\sin\theta_2}{\sin\theta_1}$$

$$mV - mV' = m_1 V_1 \cos\theta_1 + m_2 V_2 \cos\theta_2 = \frac{m_2 V_2}{\sin\theta_1} \sin\theta_2 \cos\theta_1 + m_2 V_2 \cos\theta_2$$

$$m_2 V_2 = \frac{m(v - v')}{\cos\theta_2 + \frac{\sin\theta_2 \cos\theta_1}{\sin\theta_1}} = \frac{m(v - v')\sin\theta_1}{\sin(\theta_1 + \theta_2)}$$

$$\boxed{V_2 = \frac{m}{m_2} \frac{V - V'}{\sin(\theta_1 + \theta_2)} \sin\theta_1} \qquad V_1 = \frac{m_2}{m_1} \frac{\sin\theta_2}{\sin\theta_1} V_2$$

$$\boxed{V_1 = \frac{m}{m_1} \frac{V - V'}{\sin(\theta_1 + \theta_2)} \sin\theta_2}$$

Result