EXAM II Physics 218 FALL 2018

Last Name....Key..... First..... Section Number.....

USEFUL EQUATIONS

If $f(x) = a\,x^n$, then

$$\frac{df}{dx} = n\,a\,x^{n-1}$$

$$\int f(x)\,dx = \frac{a}{n+1}\,x^{n+1} + C$$

Work – Kinetic Energy Theorem:

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{tot} \cdot d\vec{r} = \frac{1}{2}mv^2(\vec{r}_2) - \frac{1}{2}mv^2(\vec{r}_1).$$

If $\vec{F}$ is conservative, then there exists a potential energy function U such that

$$U(\vec{r}_2) - U(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

and

$$F_x = -\frac{\partial U}{\partial x}, \quad F_y = -\frac{\partial U}{\partial y}.$$

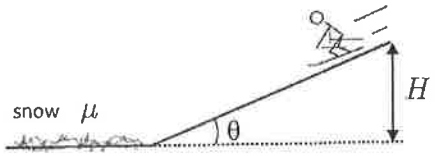
Note: The symbol g stands for the **magnitude** of the acceleration due to gravity, and therefore it is always a positive quantity.

Free-body force diagrams are very important!

Do not spend too much time on algebra!

1. (25 points) A skier of mass m slides down an icy frictionless surface starting from rest at a height H above the ground. Find the speed of the skier just before reaching the bottom of the incline. At the bottom the skier encounters level rough snow with coefficient of friction μ. How far will the skier slide along the snow before coming to rest?

Law



Application

First part of motion : only conservative forces

$\Rightarrow$ Conservation of energy $\quad PE^i + KE^i = PE^f + KE^f$

$$mgH + 0 = 0 + \frac{1}{2}mv^2$$

$$v = \sqrt{2gH}$$

Nonconservative friction force $\Rightarrow$ W-KE theorem

$$W = \Delta KE$$

$$\int_0^L -\mu mg\, dx = 0 - \frac{1}{2}mv^2$$

$$-\mu mgL = -\frac{1}{2}mv^2$$

$$\mu mgL = mgH$$

$$L = \frac{H}{\mu}$$

Result

2. (25 points) An object of mass m moves in one dimension under the influence of a single known force $\vec{F}(x) = -\alpha x^2 \hat{i}$, where α is a positive constant. Find the potential energy function associated with the force $\vec{F}$. If the object is initially located at the origin with speed v_0 in the $+x$ direction, what will be the object's speed at the location $x = d$?

Law

$$U = -\int F(x)\,dx$$

Application

$$= -\int (-\alpha x^2)\,dx$$

$$= \frac{1}{3}\alpha x^3$$

Conservation of energy $\quad U^i + KE^i = U^f + KE^f$

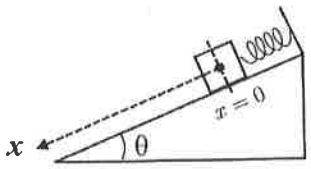
$$\frac{1}{3}\alpha(0)^3 + \frac{1}{2}mv_0^2 = \frac{1}{3}\alpha(d)^3 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 - \frac{1}{3}\alpha d^3$$

$$\Rightarrow v = \sqrt{v_0^2 - \frac{2}{3}\frac{\alpha}{m}d^3}$$

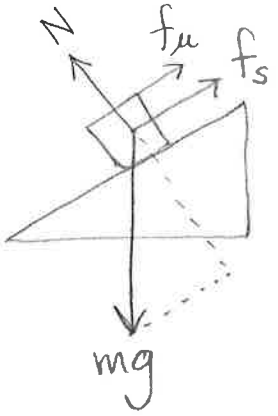
Result

3. (25 points) A block of mass m is attached to a spring with spring constant k, lying at its natural resting length at the top of a long ramp that makes an angle θ with respect to the horizontal. The coefficient of friction between the box and ramp increases down the incline as $\mu(x) = \mu_0(1+\alpha x)$, where α is a constant and x is measured from the rest length of the spring (marked with a dot in the figure). If the block is released from rest and begins sliding down the incline, how far will it travel before coming to rest? Draw the **free-body force diagram** for the block in motion.



Law

Application



Position-dependent nonconservative forces $\Rightarrow$ W-KE theorem

$$W = \Delta KE$$

$$\int_0^L (mg\sin\theta - f_s - f_\mu)\,dx = 0 - 0$$

$$\int_0^L (mg\sin\theta - Kx - \mu_0(1+\alpha x)mg\cos\theta)\,dx = 0$$

$$(mg\sin\theta)L - \tfrac{1}{2}KL^2 - \mu_0 mg\cos\theta\left(L + \tfrac{1}{2}\alpha L^2\right) = 0 \qquad \text{(divide by L)}$$

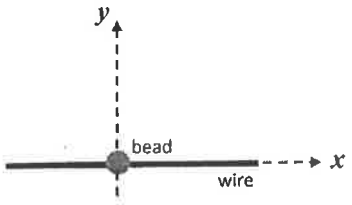
$$\Rightarrow \quad mg\sin\theta - \mu_0 mg\cos\theta = \left(\tfrac{1}{2}K + \mu_0 mg\cos\theta\,\tfrac{\alpha}{2}\right)L$$

Result

$$L = \frac{mg(\sin\theta - \mu_0\cos\theta)}{\frac{1}{2}K + \frac{\alpha}{2}\mu_0 mg\cos\theta}$$

4. (25 points) Suppose there is a conservative force with potential energy function $U(x, y) = -ax - bx^2 + 3cy^3$, where a, b, and c are positive constants, throughout the entire x-y plane. Find the associated force $\vec{F}(x, y)$. Consider a bead of mass m that is free to move along a frictionless wire that is fixed to the x axis. If the bead is released from rest at the origin, in which direction will it start to move? How fast will it be traveling after it moves a distance L?

Law



Application

$F_x = -\frac{\partial U}{\partial x} = a + 2bx$ (y treated as constant)

$F_y = -\frac{\partial U}{\partial y} = -9cy^2$ (x treated as constant)

$\vec{F} = F_x \hat{i} + F_y \hat{j}$

$\vec{F}(x=0, y=0) = (a + 2b(0))\hat{i} + (-9c(0)^2)\hat{j}$

$= a\hat{i}$ toward the right

Conservation of energy : $U^i + KE^i = U^f + KE^f$

$(-a(0) - b(0)^2 + 3c(0)^3) + 0 = (-aL - bL^2 + 3c(0)^3) + \frac{1}{2}mv^2$

$0 = -aL - bL^2 + \frac{1}{2}mv^2$

Result

$v = \sqrt{\frac{2}{m}(aL + bL^2)}$