Exam 2 Review

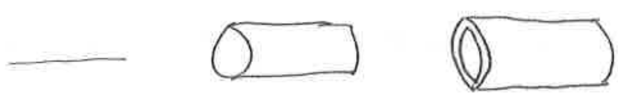
① Gauss's Law Problems

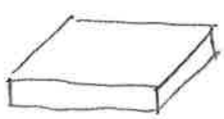
⊛ For static charge distributions

$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}}$ allows us to compute E and also $\boxed{V(b)-V(a) = -\int_a^b \vec{E}\cdot d\vec{r}}$.

⊛ Three variations

(a)  Spheres and spherical shells

(b)  Lines, cylinders, cylindrical shells

(c)  Plates

⊛ Usually left-hand side of Gauss's Law is easiest part of calculation. For the three geometries above

(a) $E(4\pi r^2)$ Gaussian sphere with radius r

(b) $E(2\pi r L)$ Gaussian cylinder with radius r and length L

(c) $E(A)$ Gaussian "cylinder" with face area A

(*) Right-hand side of Gauss's Law can be more challenging.

(i) For "uniform" charge distributions $\rho = \frac{Q_{total}}{V_{total}}$ and

$$\boxed{q_{enc} = \rho \cdot \text{Volume}_{enc}}$$

(ii) For "nonuniform" charge distributions $\rho(r)$:

$$dq_{enc} = \rho(r)\,dV$$

$$\Rightarrow \boxed{q_{enc} = \int \rho(r)\,dV}$$

(a) Spherical Gaussian surfaces $dV = 4\pi r^2\,dr$

(b) Cylindrical Gaussian surfaces $dV = 2\pi r L\,dr$

(*) Once $\vec{E}$ field is known everywhere, it is possible to compute the potential difference $\Delta V = V(b) - V(a)$ $= -\int_a^b \vec{E} \cdot d\vec{r}$ between any two points in space.

(*) Can also compute capacitance $C = \frac{Q}{\Delta V}$, where ΔV is the potential difference <u>between</u> the two conductors and Q is the magnitude of the charge on the conductors.

Exam 2 Review

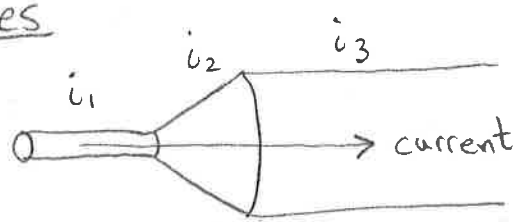
(2) Ohm's Law

(*) Crucial point is that current i is constant along the direction of flow but current density $j = \frac{i}{A}$ can vary.

Cross sectional area A.

(*) Examples

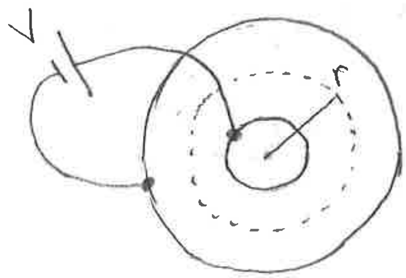
(a)



$i_1 = i_2 = i_3$

$j_1 > j_2 > j_3$

(b) Spherical shell with current flowing from inner surface to outer surface

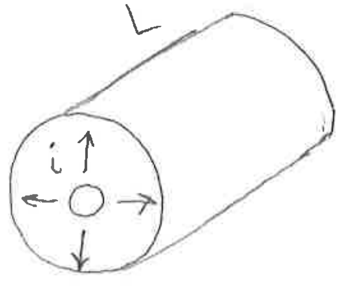


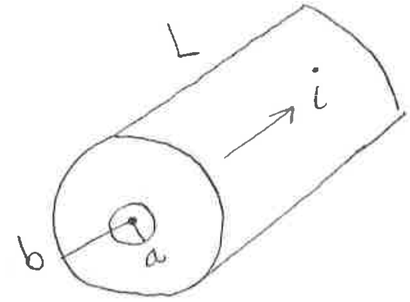
i = constant through shell of radius r

j is decreasing for larger r:

$$j = \frac{i}{4\pi r^2}$$

(c) For cylindrical shells, current can flow either from inner to outer surface or along the length of the cylinder depending on how we connect the battery





Here $j = \dfrac{i}{2\pi r L}$

(decreases as r increases)

Here $j = \dfrac{i}{\pi(b^2 - a^2)}$

(*) To calculate the resistance of an object, the current, the current density, or $\vec{E}$ field, follow these 5 steps:

(1) Identify current direction, and label current "i"

(2) Find cross sectional area A perpendicular to current flow and calculate current density $J = \dfrac{i}{A}$

(3) Find electric field from Ohm's Law: $E = \rho J$ resistivity

(4) Compute $\Delta V = -\int \vec{E} \cdot d\vec{r} = -\int \rho J \, dr$

In general either ρ or J can vary along the direction of the current: $\Delta V = -\int \rho(r) J(r) \, dr$

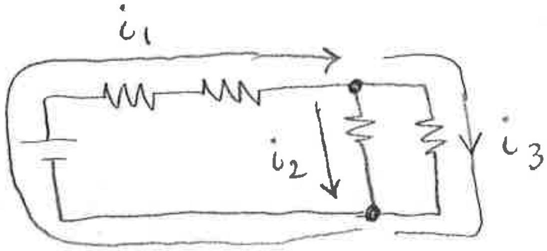
(5) Obtain R from Gauss's Law: $\Delta V = iR$

(*) Resistors in
- parallel $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$
- series $R_{eq} = R_1 + R_2$

Exam 2 Review

③ Circuits

⊛ Current i is constant along any continuous section of the circuit and only changes at a junction

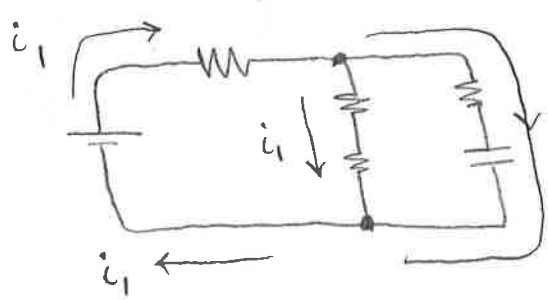


⊛ Always analyze circuits in the following steps

ⓐ Draw one current for each section of wire and label both the magnitudes ($i_1, i_2, i_3, \ldots$) and directions.

Note: choice of current direction doesn't matter

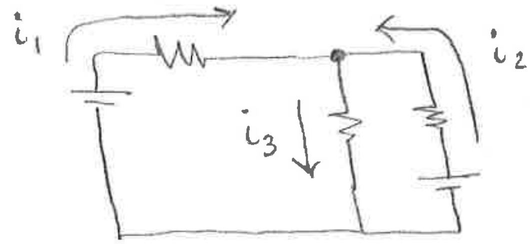
Note: If there is a capacitor in a section of wire, then after a long time charge will build up and current flow stops through the entire section



$i_2 = 0$ after a long time

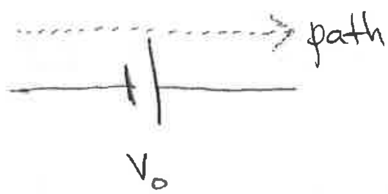
(b) At a junction, write down equation for current conservation:

$i_{in} = i_{out}$ (here choice of current direction factors in)



$i_1 + i_2 = i_3$

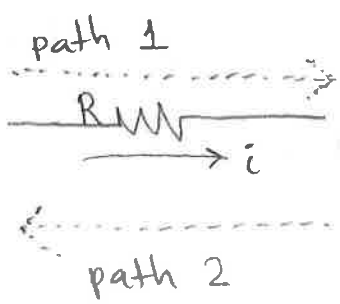
(c) Use the loop rule ($\Delta V = 0$ around closed path) as many times as needed to generate N equations for N unknowns. Calculate individual potential gains/drops across circuit elements as follows:



$+V_0$ from $-$ to $+$ terminal

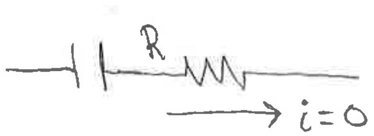
$-V_0$ from $+$ to $-$ terminal

(independent of current direction)

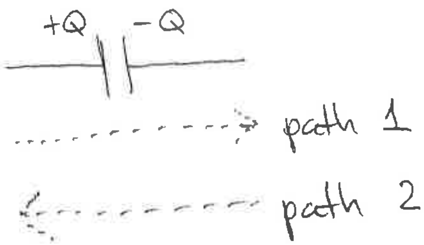


$\Delta V = -iR$ for path 1 (in direction of current)

$\Delta V = +iR$ for path 2 (where path is opposite direction of current)



$\Delta V = 0$ if current is 0 (for instance if there is a capacitor in series)

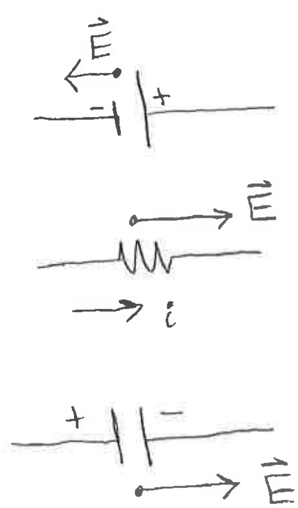


$\Delta V = -\frac{Q}{C}$ (for path 1 from + to −)

$\Delta V = +\frac{Q}{C}$ (for path 2 from − to +)

⊛ Note that your choice of path is completely independent from the current direction (you can walk with or against the current in taking a closed path)

⊛ If you are ever unsure about the sign on a potential drop/gain, think about the direction of the electric field



Moving with $\vec{E}$ field results in <u>potential drop</u> (−).

Moving against $\vec{E}$ field results in <u>potential gain</u> (+).