Exam 2 Strategies

1. Given all of the forces, you can determine an object's motion.

Case a: Time-dependent forces $F(t)$

$\Rightarrow$ Solve using Newton's 2nd Law $\vec{F}(t) = m\vec{a}(t)$

Case b: Position-dependent forces $F(x)$

$\Rightarrow$ Must solve using $W_{tot} = \int_a^b \vec{F}\cdot d\vec{r} = KE(b) - KE(a)$

Note: we can only learn how the velocity of an object depends on its position (not a complete description of its motion $\vec{x}(t)$)

2. Details for computing work

(a) Define coordinate system

(b) Write $\vec{F}(x)$ in the coordinate system $\overbrace{\vec{F} = F_x \hat{\imath}_x + F_y \hat{\imath}_y}^{\text{Usually of this form}}$.

(c) $d\vec{r}$ always equal to $dx\,\hat{\imath}_x + dy\,\hat{\imath}_y$

(d) $\vec{F}\cdot d\vec{r} = F_x\,dx + F_y\,dy$ (usually one of these vanishes)

(e) $\int_a^b$ — $b \leftarrow$ always final location; $a \leftarrow$ always initial location

Note: For springs the origin must always be chosen to be at the natural rest length of the spring.

Then $\vec{F}(x) = -K\vec{x} = -Kx\,\hat{\imath}_x$.

(3) For "conservative forces" we do not need to explicitly compute work.

Instead, define its associated potential energy function

$$U(x) = -\int F(x)\,dx$$ (no need for integration limits or integration constant)

Then if only conservative forces act on an object, energy will be conserved:

$$C = KE(a) + \underbrace{U_1(a) + U_2(a)}_{\text{may be one or many conservative forces present}} = KE(b) + U_1(b) + U_2(b)$$

(*) This is valid for any locations a and b along path.

(4) Sometimes we are nice and just give you the potential energy function $U(x)$ or $U(x,y)$.

$\Rightarrow$ Just use energy conservation to determine motion.

Force can be obtained from $F_x = -\frac{\partial U}{\partial x}$ $\quad F_y = -\frac{\partial U}{\partial y}$.

5 Does a collision or explosion take place in the question?

If "yes", then use conservation of momentum:

$$\underbrace{\vec{p}^{\,i}_{tot} = \vec{p}^{\,f}_{tot}}$$

The vector equality holds for both the x and y components separately

$$\boxed{p^i_{x,tot} = p^f_{x,tot} \quad \underline{\text{and}} \quad p^i_{y,tot} = p^f_{y,tot}}$$

(*) If we say explicitly in the question that the collision is <u>elastic</u>, then you must also enforce

$$\boxed{KE^i_{tot} = KE^f_{tot}}$$

Elastic collisions <u>only</u>.