EXAM I Physics 207 SPRING 2020

Last Name......Solutions.............. First.................................... Section Number.........

USEFUL EQUATIONS

If $f(x) = a\,x^n$, then

$$\frac{df}{dx} = n\,a\,x^{n-1}$$

$$\int f(x)\,dx = \frac{a}{n+1}\,x^{n+1} + C$$

Magnitude of electrostatic force between two point charges:

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

Definition of electric field $\vec{E}$ and electric potential V:

$$\vec{F} = q\vec{E}$$

$$U = qV$$

Relations between electric field and electric potential:

$$V(\vec{r}_2) - V(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\cdot d\vec{r}$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y}$$

Change in position vector:

$$d\vec{r} = dx\,\hat{i}_x + dy\,\hat{i}_y$$

$$d\vec{r} = dr\,\hat{i}_r + rd\theta\,\hat{i}_\theta$$

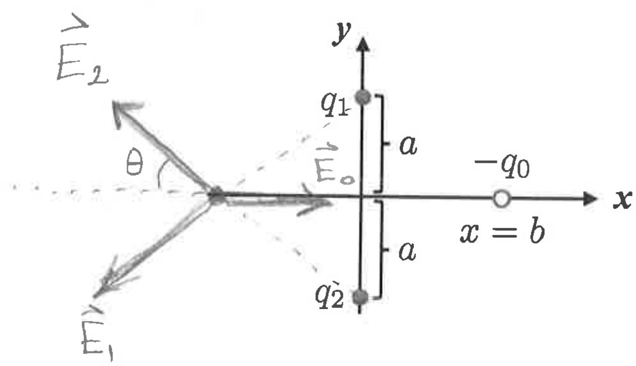
Do not spend too much time on algebra or tedious math!

Score: 1 2 3 4

1. (25 points) Consider the arrangement of three point charges $-q_0$, q_1, and q_2 shown below. What is the net electric field at the location $x = -b$ on the x axis?

Law $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

Application



Magnitudes

$$E_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_0}{(2b)^2} \rightarrow E_{0x} = \frac{1}{4\pi\varepsilon_0}\frac{q_0}{4b^2}$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a^2+b^2} \rightarrow E_{1x} = \frac{-1}{4\pi\varepsilon_0}\frac{q_1}{a^2+b^2}\cos\theta,\quad E_{1y} = \frac{-1}{4\pi\varepsilon_0}\frac{q_1}{a^2+b^2}\sin\theta$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{a^2+b^2} \rightarrow E_{2x} = \frac{-1}{4\pi\varepsilon_0}\frac{q_2}{a^2+b^2}\cos\theta,\quad E_{2y} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{a^2+b^2}\sin\theta$$

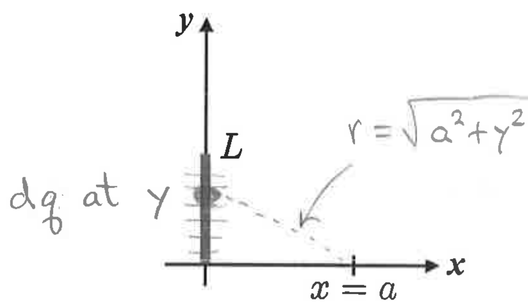
where $\cos\theta = \dfrac{b}{\sqrt{a^2+b^2}}$ and $\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$

$$E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_0}{4b^2} - \frac{q_1+q_2}{(a^2+b^2)^{3/2}}b\right]$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2-q_1}{(a^2+b^2)^{3/2}}a\right]$$

2. (25 points) Consider a charged rod that extends along the y axis from the origin to the point $y = L$. (i) If there is a charge Q uniformly distributed along the rod, find the electric potential V at the point $x = a$ on the x axis, assuming that $V(\infty) = 0$. (ii) If instead the rod has a nonuniform charge distribution $\lambda(y) = \alpha y$, find $V(x = a)$ as before. **Nontrivial integrals can be left unevaluated.**

Law $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$



Application

① $dq = \lambda\, dl = \left(\frac{Q}{L}\right) dy$

② $\int_0^L dy$

③ $dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \left(\frac{Q}{L}\, dy\right) \frac{1}{\sqrt{a^2+y^2}}$

④ $V = \int_0^L \frac{1}{4\pi\varepsilon_0} \left(\frac{Q}{L}\right) \frac{dy}{\sqrt{a^2+y^2}}$

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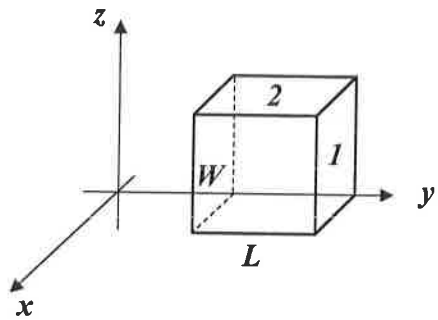
① $dq = \lambda\, dl = \alpha y\, dy$

⋮ (same as above)

④ $V = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\alpha y\, dy}{\sqrt{a^2+y^2}}$

3. (25 points) Consider a cube with edge length L whose back face lies in the yz plane and whose bottom face lies in the xy plane as shown in the figure. The cube is located a distance W along the y axis from the origin. Throughout this region of space is an electric field $\vec{E}(x,y,z) = ax^2y^2z^2\hat{i}_x + by^2\hat{i}_y + cx^2z\hat{i}_z$, where a, b, and c are constants. Find the electric flux through the right face (labeled 1) and the top face (labeled 2).

Law $\phi_E = \int \vec{E} \cdot d\vec{A}$



Application

Right

① $\hat{n} = \hat{i}_y$

② $\vec{E} \cdot \hat{n} = by^2$

③ $\vec{E} \cdot \hat{n}\Big|_{S\,(y=L+W)} = b(L+W)^2$

④ $dA = dx\,dz$

⑤ $\phi_E = \int_0^L dx \int_0^L dz\, b(L+W)^2 = bL^2(L+W)^2$

Top

① $\hat{n} = \hat{i}_z$

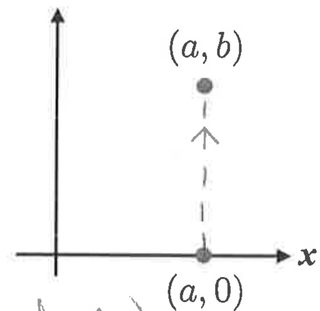
② $\vec{E} \cdot \hat{n} = cx^2$

③ $\vec{E} \cdot \hat{n}\Big|_{S\,(z=L)} = cx^2$

④ $dA = dx\,dy$

⑤ $\phi_E = \int_0^L dx \int_W^{W+L} dy\, cx^2 = \frac{1}{3}cx^3\Big|_0^L \cdot y\Big|_W^{W+L} = \frac{1}{3}cL^5$

4. (25 points) The electric field in a region of space is given by $\vec{E}(x,y) = \alpha x \hat{\imath}_x + \beta y \hat{\imath}_y$, where α and β are positive constants. (i) What is the potential difference $V(a,0) - V(a,b)$? (ii) If a charge $-q$ is placed at the location $(a,0)$ and released from rest, in what direction will it start to move (**justify your answer**)?



Law $V(\vec{r}_2) - V(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$

Application

(i)

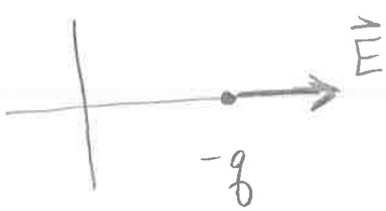
$$V(a,0) - V(a,b) = -\int_{(a,b)}^{(a,0)} (\alpha x \hat{\imath}_x + \beta y \hat{\imath}_y) \cdot (dx\, \hat{\imath}_x + dy\, \hat{\imath}_y)$$

$$= -\int_{(a,b)}^{(a,0)} \alpha x\, dx + \beta y\, dy$$

Straight upward path $\Rightarrow dx = 0$

$$\Rightarrow \Delta V = -\int_b^0 \beta y\, dy = -\frac{1}{2}\beta y^2 \Big|_b^0 = \frac{1}{2}\beta b^2$$

(ii) At $(a,0)$ the electric field is $\vec{E}(a,0) = \alpha a\, \hat{\imath}_x$



$\vec{F} = -q\vec{E}$ is in $-\hat{\imath}_x$