EXAM I Physics 206 FALL 2020

Last Name........Key........................ First.................................... Section Number..........

USEFUL EQUATIONS

If $f(x) = a\,x^n$, then

$$\frac{df}{dx} = n\,a\,x^{n-1}$$
$$\int f(x)\,dx = \frac{a}{n+1}\,x^{n+1} + C$$

For motion under **constant acceleration** a, the following formulas hold:

$$v(t) = at + v(0)$$
$$x(t) = \frac{1}{2}at^2 + v(0)t + x(0)$$
$$v^2(t_2) - v^2(t_1) = 2a\Big[x(t_2) - x(t_1)\Big]$$

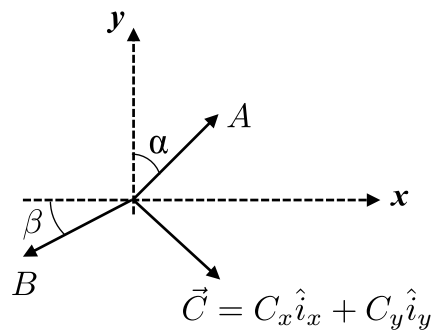
Note: The symbol g stands for the **magnitude** of the acceleration due to gravity, and therefore it is always a positive quantity.

Free-body force diagrams are very important!

Do not spend too much time on algebra!

1. (25 points) Consider three force vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. Vectors $\vec{A}$ and $\vec{B}$ have known magnitudes A and B as well as angles α and β shown in the figure. In addition, the components C_x and C_y of vector $\vec{C}$ are known. (a) What is the magnitude of vector $\vec{C}$? (b) What is the total force $\vec{F} = \vec{A} + \vec{B} + \vec{C}$?

Law



Application

$$C = \sqrt{C_x^2 + C_y^2} \quad +5$$

$$A_x = A\sin\alpha \quad +4 \qquad A_y = A\cos\alpha \quad +4$$

-2 for each sin/cos mistake

$$B_x = -B\cos\beta \quad +4 \qquad B_y = -B\sin\beta \quad +4$$

$$C_x = C_x \quad +1 \qquad C_y = C_y \quad +1$$

-2 for each sign mistakes

$$F_x = A\sin\alpha - B\cos\beta + C_x \quad +1$$

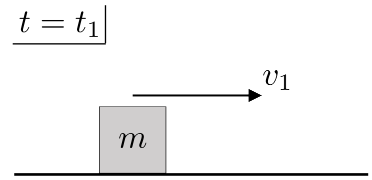
$$F_y = A\cos\alpha - B\sin\beta + C_y \quad +1$$

$$\vec{F} = F_x\hat{i}_x + F_y\hat{i}_y$$

Result

2. (25 points) The acceleration of a block of mass m moving along a straight line is given by $a(t) = -c_1 t^2$ where c_1 is a positive constant. At time $t = 0$, the object's position is $x = 0$ and at time $t = t_1$ its velocity is measured to be v_1, where v_1 is a positive constant. (a) Find the object's position as a function of time. (b) Find the time t_r at which the object reverses its direction of motion.

Law



Application

+4

$$v(t) = \int a(t)\,dt = \int -c_1 t^2\,dt = -\frac{1}{3}c_1 t^3 + C$$

+3

$$v(t_1) = v_1 \Rightarrow -\frac{1}{3}c_1 t_1^3 + C = v_1$$

$$\Rightarrow C = \frac{1}{3}c_1 t_1^3 + v_1$$ +2

+4

$$\Rightarrow v(t) = -\frac{1}{3}c_1 t^3 + \frac{1}{3}c_1 t_1^3 + v_1$$ +2

$$x(t) = \int v(t)\,dt = -\frac{1}{12}c_1 t^4 + \left(\frac{1}{3}c_1 t_1^3 + v_1\right)t$$ +4

$$\boxed{v(t_r) = 0} \Rightarrow -\frac{1}{3}c_1 t^3 + \frac{1}{3}c_1 t_1^3 + v_1 = 0$$

+3

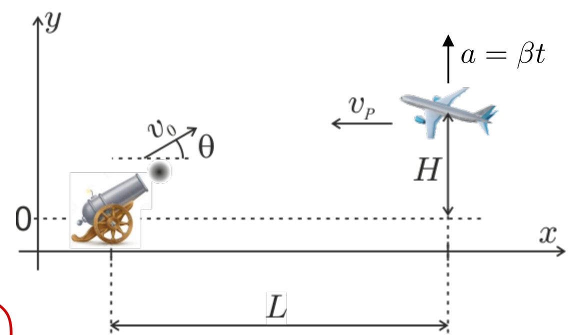
$$t^3 = t_1^3 + \frac{3}{c_1}v_1$$ +1

0 integration constant

+2

Result

3. (25 points) At time $t = 0$ a cannon ball located at the origin is fired with adjustable initial speed v_0 and angle θ_1 toward an airplane located at $x = L$ and $y = H$ with horizontal speed v_p toward the cannon. At time $t = 0$ the airplane begins to accelerate straight upward with magnitude $a = \beta t$ as shown in the figure. Write down a sufficient number of equations (**but do not solve!**) that in principle could be used to find the values of v_0 and θ_1 needed to hit the airplane.



Law

Cannon ball

Application

$a_y = -g$ +2 $\quad a_x = 0$ +2

$v_{y0} = v_0 \sin\theta \quad v_{x0} = v_0 \cos\theta$

$y_0 = 0$ +2 $\quad x_0 = 0$

+2 +2

$y_c(t) = v_0 \sin\theta\, t - \frac{1}{2} g t^2 \qquad x_c(t) = v_0 \cos\theta\, t$

Plane $\quad a_y = \beta t \qquad a_x = 0$

$v_{y0} = 0$ +2 $\qquad v_{x0} = -v_p$ +2

$y_0 = H \qquad x_0 = L$ +2

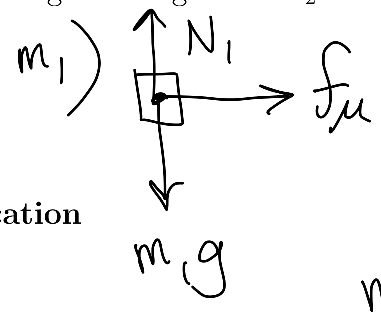
$v_p(t) = \frac{1}{2}\beta t^2$ +2 $\qquad x_p(t) = L - v_p t$ +2

$y_p(t) = H + \frac{1}{6}\beta t^3$ +2

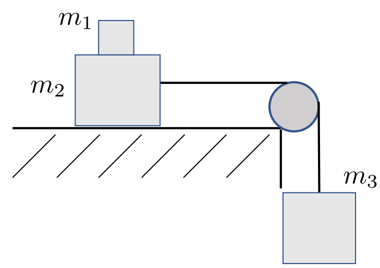
Result $\quad y_c(t) = y_p(t)$ +3 $\quad x_c(t) = x_p(t)$

4. (25 points) Three blocks with masses m_1, m_2, and m_3 are connected by an unstretchable string by a pulley as shown in the figure. The string and pulley have negligible mass. There is no friction between block m_2 and the ground, but there is friction between blocks m_1 and m_2 with known coefficient of friction μ. After being released from rest, block m_3 falls downward while blocks m_1 and m_2 move together to the right. (a) Draw a separate free-body diagram for each block. (b) Find the magnitude of the acceleration for the system. (c) Find the magnitude of the friction force acting on block m_1. (d) If an additional external force F_0 pulls m_3 downward, what value of F_0 would cause block m_1 to begin sliding off of m_2?

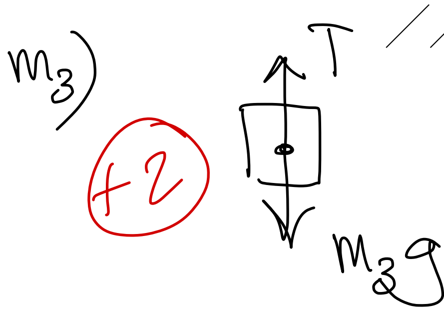
Law



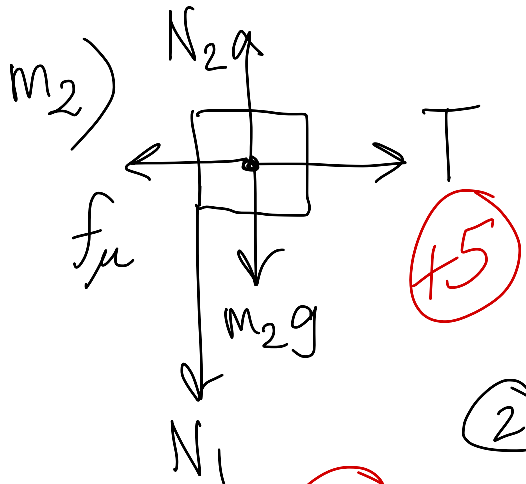
+3



Application



+2



+5

+3 +2

② $T - f_\mu = m_2 a$ ③ $m_3 g - T = m_3 a$

$T = m_3(g-a)$

① $f_\mu = m_1 a$ +2 $m_3 g - f_\mu = (m_2+m_3)a$

$m_3 g - m_1 a = (m_2+m_3)a$

+1 +1

$\boxed{f_\mu = \frac{m_1 m_3}{m_1+m_2+m_3} g}$ $\Rightarrow$ $\boxed{a = \frac{m_3}{m_1+m_2+m_3} g}$

+2 $f_\mu = \mu N$ $T - f_\mu = m_2 a$ $F_0 + m_3 g - T = m_3 a$ +1

Result

+1 +1

$\mu m_1 g = m_1 a$ $T = m_2 \mu g + \mu m_1 g$
$= (m_1+m_2)\mu g$

$\boxed{F_0 = (m_1+m_2+m_3)\mu g - m_3 g}$ +1