Exam 1 Review

(*) Always start each problem by identifying the question type:

(1) Coulomb's Law for point charges

(2) Electric field from charge distribution

(3) Electric potential from the electric field or from a charge distribution

(4) Electric field from the electric potential

(5) Electric flux and Gauss's Law

(*) Coulomb's Law for point charges

To find the net force on one charge due to other nearby charges, follow these steps:

1. Draw the individual force vectors on the charge in question due to all other charges
2. Compute the magnitudes of each individual force according to $F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$

③ Break up each force vector into its x and y components (be sure to account for ± signs!)

④ Add separately all x-components and y-components to obtain $\vec{F} = F_x \hat{\imath}_x + F_y \hat{\imath}_y$

Note that you can compute the electric field at a point in space due to a set of point charges in exactly the same way, except $E = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}$.

Fundamental relationship between the two:

$$\vec{F} = q\vec{E}$$

⊛ Electric field due to charge distribution

To find the electric field due to a linear charge distribution, follow these steps:

① Break up physical line charge into small segments of length $ds = \{dx, dy, R\,d\theta\}$ and compute

$$dq = \lambda(s)\,ds = \{\lambda(x)\,dx,\ \lambda(y)\,dy,\ \lambda(\theta)\,R\,d\theta\}$$

Note that the charge distribution might be explicitly given, $\lambda(x) = \alpha x$, or you may need to compute it for uniform charge distributions, e.g., $\lambda = \frac{Q}{L}$ or $\lambda = \frac{Q}{2\pi R}$ (for a full circle)

2. Define the integration region that <u>must</u> be over the physical length of the charge distribution, e.g. $\int_0^L dy$.

3. Choose arbitrary point s within the integration region and compute

$$dE(s) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{dq(s)}{r^2(s)}$$

(Both dq and r can depend on s.)

4. Break up $d\vec{E}$ into its x and y components.

Look for symmetries that might cause one of these components to vanish.

5. Integrate x and y components separately to obtain

$$E_x = \int dE_x \qquad E_y = \int dE_y$$

⊛ Electric potential from electric field

Given the electric field $\vec{E}(\vec{r})$ we can compute the potential difference between two positions:

$$V(\vec{r}_2) - V(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$$

We can also compute the electric potential function:

$$V(\vec{r}) = V(\vec{r}) - V(\infty) = -\int_{\infty}^{\vec{r}} \vec{E} \cdot d\vec{r} \qquad (\text{choosing } V(\infty) \equiv 0)$$

If $\vec{E}(\vec{r})$ is given in Cartesian coordinates, use

$$d\vec{r} = dx\, \hat{i}_x + dy\, \hat{i}_y$$

If $\vec{E}(\vec{r})$ is given in polar coordinates, use

$$d\vec{r} = dr\, \hat{i}_r + r\,d\theta\, \hat{i}_\theta$$

If $\vec{E}(\vec{r})$ is given piecewise (it has a different form in different regions of space), you will have to break up the integration region

Fundamental relation between V and potential energy U:

$$U = qV.$$

To calculate electric potential function near a charge distribution, follow same steps as electric field calculation, except

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

and there is no need to break up into components since V is a scalar quantity.

⊛ Electric field from electric potential

Given the electric potential in Cartesian coordinates $V(x,y)$, we compute

$$E_x = -\frac{\partial V}{\partial x} \quad \text{and} \quad E_y = -\frac{\partial V}{\partial y}$$

Given a radial electric potential $V(r)$, we compute

$$E_r = -\frac{\partial V}{\partial r}$$

⊛ Electric flux

Given an electric field $\vec{E}$ and a surface S, we compute the electric flux Φ_E (a scalar quantity) through S as follows:

$$\phi_E = \int_S \vec{E} \cdot d\hat{A} = \int_S (\vec{E} \cdot \hat{n})\, dA$$

① Start by finding $\hat{n}$ for the surface

② Determine $\vec{E}$ in vector notation and evaluate

$\vec{E} \cdot \hat{n} = f(x, y, z)$ ← result will be a function of x, y, z

③ Evaluate $\vec{E} \cdot \hat{n} = f(x, y, z)$ on the surface S

④ Determine dA, which must have units of area and is often one of the following $dA = dx\,dy$, $dy\,dz$, or $dx\,dz$

⑤ Determine integration bounds and integrate

Note that if you integrate a constant, you obtain

$$\phi_E = \int c\, dA = c \int dA = CA$$

← surface area

regardless of integration variables.