Exam 1 Pro Tips

1. Identify the "type" of problem being asked

All of the questions will be variations of the following:

(i) Motion under constant acceleration

(ii) Motion with time-dependent acceleration, e.g., $a(t) = Kt^3$.

Always ask yourself: "Is acceleration constant or time-dependent?"

(iii) Vector algebra

(iv) Newton's Laws (2nd and 3rd laws)

Note: Remember that projectile motion or two-dimensional motion is nothing more than 2 independent one-dimensional motions

2. Motion under constant acceleration (by "constant" we mean constant in time)

(*) Sometimes it is not obvious whether acceleration is constant or not:

$x(t) = 3t^2 + 5t$

$\downarrow$

$v(t) = 6t + 5$

$\downarrow$

$a(t) = 6 \rightarrow$ Yes!

$v(t) = \alpha t^2$

$\downarrow$

$a(t) = 2\alpha t$

$\downarrow$

No!

$a(t) = \alpha$

$\downarrow$

Yes!

- If acceleration is constant, then we can use any of the following:

$$v(t) = at + v(0)$$

$$x(t) = \frac{1}{2}at^2 + v(0)t + x(0)$$

$$v^2(t_2) - v^2(t_1) = 2a\left[x(t_2) - x(t_1)\right]$$

BE VERY CAFEFUL WITH $\pm$ SIGNS on $x(0)$, $v(0)$, a, etc.

- Objects in freefall have constant acceleration g near the Earth's surface (g always positive, so normally $a = -g$)

3) Motion under nonconstant acceleration

- In this case we must integrate or differentiate to go from one kinematic variable to another

Given:

$\boxed{a(t)}$

$\downarrow$

$$v(t) = \int a(t)\,dt + C_1$$

$\downarrow$

$$x(t) = \int v(t)\,dt + C_2$$

$\boxed{v(t)}$

$\downarrow$

$$a(t) = \frac{dv}{dt}$$

$$x(t) = \int v(t)\,dt + C$$

$\boxed{x(t)}$

$\downarrow$

$$v(t) = \frac{dx}{dt}$$

$\downarrow$

$$a(t) = \frac{dv}{dt}$$

Exam 1 Pro Tips

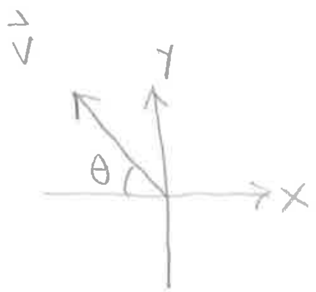
* Integration constants C determined from relevant "initial conditions" (values of the kinematic variables v and x at specific points in time)
* Be mindful of implicit initial conditions:
  (1) Drop object at $t=0 \Rightarrow v(0)=0$
  (2) object reverses direction at $t_r \Rightarrow v(t_r)=0$
* Collisions between objects 1 and 2: very simple $\rightarrow$ just find $x_1(t)$, $y_1(t)$ and set equal to $x_2(t)$, $y_2(t)$. Effectively, just 4 one-dimensional problems.

4 Vector Addition

* Always break up vectors into their x and y components. Then add all x components to get V^x_{total} and all y components to get V^y_{total}:

$$\vec{V}_{total} = \underbrace{V_x^{total}\,\hat{\imath} + V_y^{total}\,\hat{\jmath}}_{\text{Be sure your final answer IS a vector by including } \hat{\imath} \text{ and } \hat{\jmath} \text{ unit vectors.}} \qquad \begin{bmatrix} \hat{\imath} = \hat{\imath}_x \\ \hat{\jmath} = \hat{\imath}_y \end{bmatrix} \text{ Equivalent notation.}$$

* I recommend calculating the magnitudes of the vector components first (using basic geometry), and then determining signs by inspection:



magnitude direction

$$V_x = (v\cos\theta)(-\hat{\imath}) = -v\cos\theta\,\hat{\imath}$$

$$V_y = (v\sin\theta)(\hat{\jmath}) = v\sin\theta\,\hat{\jmath}$$

(5) Newton's Laws

* Always start by drawing one free-body diagram for each object that may undergo motion

* Usually for each object there will be two "equations of motion":

$$\left.\begin{array}{l} \text{(i) } F_x^{total} = m a_x \\ \text{(ii) } F_y^{total} = m a_y \end{array}\right\}$$

Again, always be careful with signs when adding forces!

* Remember that all forces come in equal and opposite pairs (but acting on different objects). So, if there are multiple objects interacting, be sure to identify any of these paired forces.

* Questions involving friction: Always ask yourself if the two surfaces are sliding (literally moving) against each other!

Exam 1 Pro Tips

* Static friction forces: they adjust to whatever is needed to keep object from sliding (no formula for f_μ). Only at instant of first sliding is $f_\mu = \mu_s N$.

* Kinetic friction: sliding objects always experience a frictional force $f_\mu = \mu_K N$.

* Tension forces: act equally but in opposite directions at the two ends of the rope

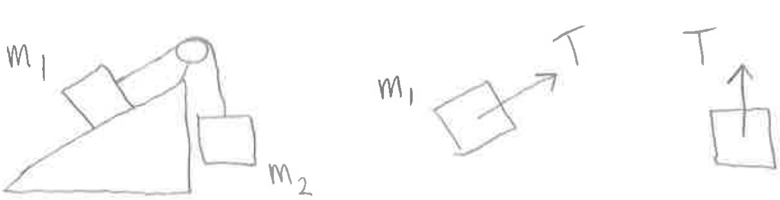
Never forget Newton's 3rd Law pairs (same magnitudes!):

(a) 

(Normal forces)

(b) 

(Friction forces)

(c) 

(Tension forces)